1. Find each of the following antiderivatives.

(a) \( \int \frac{\cos(\theta)}{4} \, d\theta \)

(b) \( \int (4-9x+x^2) \, dx \)

(c) \( \int (12e^x + \sin(x)) \, dx \)

(d) \( \int (6y - y^3)^2 \, dy \)

(e) \( \int (86t^7 - \sqrt[3]{t}) \, dt \)

(f) \( \int \frac{3t^3 - 6\sqrt{t} - \frac{9}{t}}{t} \, dt \)

(g) \( \int \left( \frac{1}{u} - \frac{1}{u^2} \right) \left( 2 + \frac{3}{\sqrt{u}} \right) \, du \)

Solution

(a) Use trigonometric derivative rules backwards.

\[
\int \frac{\cos(\theta)}{4} \, d\theta = \frac{\sin(\theta)}{4} + C
\]

(b) Use power rule backwards.

\[
\int (4-9x+x^2) \, dx = 4x - \frac{9}{2}x^2 + \frac{1}{3}x^3 + C
\]

(c) Use exponential and trigonometric derivative rules backwards.

\[
\int (12e^x + \sin(x)) \, dx = 12e^x - \cos(x) + C
\]

(d) Expand the integrand, then antidifferentiate.

\[
\int (6y - y^3)^2 \, dy = \int (36y^2 - 12y^4 + y^6) \, dy = 12y^3 - \frac{12}{5}y^5 + \frac{1}{7}y^7 + C
\]

(e) Use power rule backwards.

\[
\int (86t^7 - \sqrt[3]{t}) \, dt = \frac{86}{8}t^8 - \frac{3}{4}t^{4/3} + C
\]

(f) Write the integrand as a sum of power functions then antidifferentiate.

\[
\int \frac{3t^3 - 6\sqrt{t} - \frac{9}{t}}{t} \, dt = \int \left( 3t^2 - 6t^{-1/2} - 9t^{-2} \right) \, dt = t^3 - 12t^{1/2} + 9t^{-1} + C
\]

(g) Expand the integrand, then antidifferentiate.

\[
\int \left( \frac{1}{u} - \frac{1}{u^2} \right) \left( 2 + \frac{3}{\sqrt{u}} \right) \, du = \int \left( 2 + 3u^{-1/2} - 2u^{-1} - 3u^{-3/2} \right) \, du = 2u + 6u^{1/2} - 2\ln(|u|) + 6u^{-1/2} + C
\]
2. The marginal revenue of a certain commodity is

\[ MR(x) = -9x^2 + 24x + 48 \]

Find the price that maximizes total revenue. (Assume that \( R(0) = 0 \).)

**Solution**

Revenue is maximized when \( MR(x) = 0 \) (since \( MR(x) = R'(x) \)).

\[
0 = MR(x) = -9(3x + 4)(x - 4) \implies x = 4
\]

So revenue is maximized when \( x = 4 \). To find the price, we first need to find the total revenue, which we obtain by antidifferentiation.

\[
R(x) = \int MR(x) \, dx = \int (-9x^2 + 24x + 48) \, dx = -3x^3 + 12x^2 + 48x + C
\]

Since \( R(0) = 0 \), we find that \( C = 0 \). So the total revenue is

\[
R(x) = -3x^3 + 12x^2 + 48x
\]

Since revenue is generally \( R(x) = xp(x) \), it follows that the price is

\[
p(x) = \frac{R(x)}{x} = -3x^2 + 12x + 48
\]

Hence the price that maximizes the revenue is \( p(4) = 48 \).

3. A particle moves along the \( x \)-axis in such a way that its acceleration at time \( t > 0 \) is

\[ a(t) = 1 - \frac{1}{t^2} \]

The particle’s velocity at time \( t = 2 \) is \( v(2) = 5.5 \). What is the net distance the particle travels between the times \( t = 3 \) and \( t = 6 \)?

**Solution**

First we find the particle’s velocity by anti-differentiating \( a(t) \).

\[
v(t) = \int a(t) \, dt = \int \left(1 - t^{-2}\right) \, dt = t + t^{-1} + C
\]

Now we find the value of \( C \) by using the fact that \( v(2) = 5.5 \).

\[
5.5 = 2 + \frac{1}{2} + C \implies C = 3
\]

Hence the velocity of the particle is

\[
v(t) = t + \frac{1}{t} + 3
\]
Now we find the position of the particle by anti-differentiating $v(t)$.

$$x(t) = \int v(t) \, dt = \int \left( t + \frac{1}{t} + 3 \right) \, dt = \frac{1}{2}t^2 + \ln(|t|) + 3t + C$$

The value of $C$ is not needed since we are only interested in a difference of position. The net distance traveled between $t = 3$ and $t = 6$ is

$$\Delta x = x(6) - x(3) = \left( \frac{1}{2} \cdot 36 + \ln(6) + 18 + C \right) - \left( \frac{1}{2} \cdot 9 + \ln(3) + 9 + C \right) = 22.5 + \ln(2)$$