1. The total cost of producing \( x \) widgets is
\[
C(x) = x^3 + 9x^2 + 18x + 200
\]
and the selling price per unit is
\[
p(x) = 45 - 2x^2
\]
What is the optimal price? (That is, what price maximizes total profit?)

2. Suppose the total cost of producing \( x \) units is
\[
C(x) = 2x^4 - 10x^3 - 18x^2 + x + 5
\]
Find the smallest and largest values of marginal cost for \( 0 \leq x \leq 5 \).

3. Suppose the total cost of manufacturing \( x \) widgets is
\[
C(x) = 3x^2 + 5x + 75
\]
What level of production minimizes the average cost per unit?

4. The value of a piece of land \( t \) years from now (in the dollars of that year) is
\[
Q(t) = Q_0 t^{0.15} e^{0.2\sqrt{t}}
\]
where \( Q_0 = 100,000 \). The prevailing effective annual interest rate is 5%, compounded continuously. How many years from now is the optimal time to sell the land? (That is, how many years from now is the present value of the land a maximum?)

5. A tour agency is booking a tour and has 100 people signed up. The price of a ticket is $2000 per person. The agency has booked a plane seating 150 people at a cost of $125,000. Additional costs to the agency are incidental fees of $500 per person. For each $10 that the price is lowered, a new person will sign up. How much should the price be lowered for all participants to maximize the profit to the tour agency?