1. The sum of two numbers is 80. Find the largest possible product.

2. The sum of two numbers is 10. Find the smallest possible value for the sum of their squares.

3. Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 4, assuming that one side of the rectangle lies on the diameter of the semicircle.

4. Find the dimensions of the rectangle of largest area whose lower vertices lie on the \( x \)-axis and whose upper vertices lie on the graph of \( y = e^{-x^2} \).

5. A farmer is constructing a rectangular fence on a straight river. The side of the rectangle bordering the river does not need any fencing. If the farmer has 1000 feet of fencing, what is the largest possible area he may enclose?

6. A farmer with 1600 feet of fencing wants to enclose a rectangular area and then divide it into four equal-area pens with fencing parallel to one side of the rectangle. What is the largest possible area that a single pen can enclose?

7. A truck is 250 miles east of a sports car and is traveling west at a constant speed of 60 miles per hour. Meanwhile, the sports car is going north at 80 miles per hour. When will the truck and car be closest to each other? What is the minimum distance between them?

8. Suppose we want to construct a rectangular aquarium that must hold a volume of 4000 in\(^3\). The length of the base will be twice the width of the base. The top and bottom bases of the tank cost $1.50/in\(^2\). Each of the sides of the tank costs $3/in\(^2\). Find the dimensions (length, width, height) of the cheapest tank.