1. For each function, do all of the following.

- Find all vertical asymptotes and all horizontal asymptotes.
- Find all first-order critical numbers.
- Find where the function is increasing and where the function is decreasing.
- Classify each critical value as a relative maximum, relative minimum, or neither.
- Find all second-order critical numbers.
- Find where the graph of \( y = f(x) \) is concave up and where it is concave down.
- Identify any inflection points.
- Sketch the graph of \( y = f(x) \).

(a) \( f(x) = \frac{1}{3}x^3 - 9x + 2 \)

(b) \( f(x) = (x + 1)^2(x - 5) \)

(c) \( f(x) = \frac{x}{x^2 + 1} \)

(d) \( f(x) = x - \sin(2x) \) on \([0, \pi]\)

(e) \( f(x) = 1 + 2x + 18x^{-1} \)

(f) \( f(x) = 1 - \frac{x}{4 - x} \)

(g) \( f(x) = \sqrt{x^3 - 48x} \)

(h) \( f(x) = \ln(4 - x^2) \)

2. Sketch the graph of a function \( f \) that satisfies all of the following conditions.

- \( f'(x) > 0 \) when \( x < 2 \) and when \( 2 < x < 5 \)
- \( f'(x) < 0 \) when \( x > 5 \)
- \( f'(2) = 0 \)
- \( f''(x) < 0 \) when \( x < 2 \) and when \( 4 < x < 7 \)
- \( f''(x) > 0 \) when \( 2 < x < 4 \) and when \( x > 7 \)

3. For each part, calculate the limit or show that it does not exist. Show all work.

(a) \( \lim_{x \to \infty} \left( \frac{3x - 5}{x + 1} \right) \)

(b) \( \lim_{x \to -\infty} \left( \frac{3x}{\sqrt{4x^2 + 9}} \right) \)

(c) \( \lim_{x \to 0^+} \left( \frac{x^2 - x + 4}{2x + \sin(x)} \right) \)

(d) \( \lim_{x \to \infty} \left( \frac{(x - 3)(2x + 4)(x - 5)}{(3x + 1)(4x - 7)(x + 2)} \right) \)

(e) \( \lim_{x \to -\infty} \left( \frac{(x - 3)(2x + 4)(x - 5)}{(3x + 1)(4x - 7)(x + 2)} \right) \)

(f) \( \lim_{x \to 3^-} \left( \frac{2x^2 + 8}{x^2 - 9} \right) \)
4. Consider the function
\[ f(x) = \frac{(x - 1)(2x + 5)}{(x + 1)(3x - 6)} \]
(a) Find all horizontal asymptotes of \( f \), if any.
(b) Find all vertical asymptotes of \( f \), if any.
(c) At each vertical asymptote of \( f \), find both corresponding one-sided limits.

5. Consider the function
\[ f(x) = \frac{2e^x + 3}{1 - e^x} \]
(a) Find all horizontal asymptotes of \( f \), if any.
(b) Find all vertical asymptotes of \( f \), if any.
(c) At each vertical asymptote of \( f \), find both corresponding one-sided limits.

6. Sketch the graph of a function \( f \) that satisfies all of the following conditions.
   - the lines \( y = 1 \) and \( x = 3 \) are asymptotes
   - \( f \) is increasing for \( x < 3 \) and \( 3 < x < 5 \), and \( f \) is decreasing elsewhere
   - the graph of \( y = f(x) \) is concave up for \( x < 3 \) and for \( x > 7 \)
   - the graph of \( y = f(x) \) is concave down for \( 3 < x < 7 \)
   - \( f(0) = f(5) = 4 \) and \( f(7) = 2 \)