1. Determine all points where the following function is continuous. 
*Make sure you give a justification for any x-value at which you claim f is continuous.*

\[ f(x) = \begin{cases} 3x^2 - x + 1, & x < -2 \\ 15 + \sin(2\pi x), & -2 \leq x < 3 \\ 2x - 4, & 3 \leq x \end{cases} \]

2. Let \( f(x) = \frac{x^3 - 9x}{x + 3} \).
   
   (a) What is the domain of \( f \)?
   
   (b) Find all points where \( f \) is discontinuous.
   
   (c) For each \( x \)-value you found in part (b), determine what value should be assigned to \( f \), if any, to guarantee that \( f \) will be continuous there.
   
   *(For example, if you claim \( f \) is discontinuous at \( x = a \), then you should determine the value that should be assigned to \( f(a) \), if any, to guarantee that \( f \) will be continuous at \( x = a \).)*

3. Let \( f(x) = \frac{\sqrt{2x^2 + 1} - 1}{x^2(x - 3)} \).
   
   (a) What is the domain of \( f \)?
   
   (b) Find all points where \( f \) is discontinuous.
   
   (c) For each \( x \)-value you found in part (b), determine what value should be assigned to \( f \), if any, to guarantee that \( f \) will be continuous there.
   
   *(For example, if you claim \( f \) is discontinuous at \( x = a \), then you should determine the value that should be assigned to \( f(a) \), if any, to guarantee that \( f \) will be continuous at \( x = a \).)*

4. Find the values of the constants \( a \) and \( b \) that make the following function continuous for all real numbers.

   \[ f(x) = \begin{cases} ax^2 - x, & x < 4 \\ 6, & x = 4 \\ x^3 + bx, & x > 4 \end{cases} \]

5. Find the values of the constants \( a \) and \( b \) that make the following function continuous for all real numbers. *You may assume \( a > 0 \).*

   \[ f(x) = \begin{cases} \frac{1 - \cos(ax)}{x^2}, & x < 0 \\ 2a + b, & x = 0 \\ \frac{x^2 - bx}{\sin(x)}, & x > 0 \end{cases} \]

6. Prove that the equation \( \sqrt{x} + x^3 = 1 \) has a solution in the interval \([0, 1]\).

7. Prove that the equation \( x^4 + 3x^2 + 2 = 4x^3 + 8x \) has a solution.