1. Evaluate the following using the given graph.

![Graph]

(a) \( \lim_{x \to -3^-} f(x) \) 
(b) \( \lim_{x \to -3^+} f(x) \) 
(c) \( \lim_{x \to -3} f(x) \) 
(d) \( f(-3) \) 
(e) \( \lim_{x \to -2^-} f(x) \) 
(f) \( \lim_{x \to -2^+} f(x) \) 
(g) \( \lim_{x \to -2} f(x) \) 
(h) \( f(-2) \) 
(i) \( \lim_{x \to 1^-} f(x) \) 
(j) \( \lim_{x \to 1^+} f(x) \) 
(k) \( \lim_{x \to 1} f(x) \) 
(l) \( f(1) \) 
(m) \( \lim_{x \to 2^-} f(x) \) 
(n) \( \lim_{x \to 2^+} f(x) \) 
(o) \( \lim_{x \to 2} f(x) \) 
(p) \( f(2) \)

Solution

(a) \( \lim_{x \to -3^-} f(x) = -2 \) 
(b) \( \lim_{x \to -3^+} f(x) = -2 \) 
(c) \( \lim_{x \to -3} f(x) = -2 \) 
(d) \( f(-3) = -2 \) 
(e) \( \lim_{x \to -2^-} f(x) = -5 \) 
(f) \( \lim_{x \to -2^+} f(x) = 0 \) 
(g) \( \lim_{x \to -2} f(x) \) DNE 
(h) \( f(-2) = 2 \) 
(i) \( \lim_{x \to 1^-} f(x) = 4 \) 
(j) \( \lim_{x \to 1^+} f(x) = -4 \) 
(k) \( \lim_{x \to 1} f(x) \) DNE 
(l) \( f(1) \) DNE 
(m) \( \lim_{x \to 2^-} f(x) = -4 \) 
(n) \( \lim_{x \to 2^+} f(x) = \infty \) 
(o) \( \lim_{x \to 2} f(x) \) DNE 
(p) \( f(2) = -4 \)

2. Suppose \( \lim_{x \to 0} (f(x) + g(x)) \) exists. Is it true that \( \lim_{x \to 0} f(x) \) and \( \lim_{x \to 0} g(x) \) also exist? Explain your answer.

Solution

No. Let \( f(x) \) be any function such that \( \lim_{x \to 0} f(x) \) does not exist. (For example, \( f(x) = \frac{|x|}{x} \)). Let \( g(x) = -f(x) \). Then it is trivial that

\[
\lim_{x \to 0} (f(x) + g(x)) = \lim_{x \to 0} (f(x) - f(x)) = \lim_{x \to 0} (0) = 0
\]
Hence \( \lim_{x \to 0} (f(x) + g(x)) \) exists but neither \( \lim_{x \to 0} f(x) \) nor \( \lim_{x \to 0} g(x) \) exists.

3. Suppose \( \lim_{x \to 2} \left( \frac{f(x) - 3}{x - 2} \right) = 5 \) and \( \lim_{x \to 2} f(x) \) exists (and is equal to \( f(2) \)). What is the value of \( f(2) \)?

**Solution**

Direct substitution of \( x = 2 \) in the limit gives \( \left( \frac{f(2) - 3}{0} \right) \). If \( f(2) \) were anything other 3, this would give us a limit of the form \( \frac{c}{0} \) where \( c \neq 0 \). This would mean that the limit could not exist. (We will study infinite limits in more detail in chapter 4.)

The only possible way that the given limit could exist (we know it exists because it is equal to 5) while still having the division by 0 from direct substitution is if there were actually some cancellation in the numerator \( f(x) - 3 \). That is, we would need \( f(2) = 3 \) for direct substitution to give us \( \frac{0}{0} \). Hence \( f(2) = 3 \).