1. Find an equation of the line tangent to the graph of

\[ x^3 + y^3 = y + 21 \]

at the point \((3, -2)\).

**Solution**

Differentiate the given equation implicitly with respect to \(x\).

\[ 3x^2 + 3y^2 \cdot y' = y' \]

Substitute \(x = 3\) and \(y = -2\), then solve for \(y'\).

\[ 3(3)^2 + 3(-2)^2 \cdot y' = y' \]
\[ 27 + 12y' = y' \]
\[ y' = -\frac{27}{11} \]

Thus an equation of the desired tangent line is

\[ y - (-2) = -\frac{27}{11}(x - 3) \]

2. A ladder 13 feet long rests against a vertical wall and is sliding down the wall at the rate of 3 feet per second at the instant the foot of the ladder is 5 feet from the base of the wall. At this instant, how fast is the foot of the ladder moving away from the wall?

*You must include correct units as part of your answer.*

**Solution**

Let \(x\) be the distance from the foot of the ladder to the wall. Let \(y\) be the distance of the top of the ladder to the ground. Then \(x\) and \(y\) satisfy the equation

\[ x^2 + y^2 = 13^2 \]

Differentiating with respect to time gives

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]

Each of these equations holds for all time. Now we substitute the given information to find equations that hold at the desired instant. So we substitute \(\frac{dy}{dt} = -3\) and \(x = 5\). We obtain the following equations.

\[ 25 + y^2 = 169 \]
\[ 10 \frac{dx}{dt} - 6y = 0 \]
Our goal is to find \( \frac{dx}{dt} \). Solving the first equation for \( y \) gives \( y = 12 \). Substituting \( y = 12 \) into the second equation gives

\[
10 \frac{dx}{dt} - 72 = 0
\]

We find that \( \frac{dx}{dt} = 7.2 \). Hence at the desired instant, the foot of the ladder is moving away from the wall at a rate of 7.2 feet per second.