1. At a certain factory, the total cost (in dollars) of manufacturing \( q \) tables during the daily production run is

\[
C(q) = 0.2q^2 + 10q + 900
\]

From experience, it has been determined that approximately

\[
q(t) = t^2 + 99t
\]

tables are manufactured during the first \( t \) hours of a production run. **Make sure to indicate the units of your answer in each question below.**

(a) Calculate \( C'(50) \) and explain its precise meaning.

(b) Compute the rate at which the total manufacturing cost is changing with respect to time one hour after production begins.

**Solution**

(a) We have that \( C'(q) = 0.4q + 10 \), whence \( C'(50) = 0.4 \cdot 50 + 10 = 20 \) with units of \$/table (dollars per table). This means that at the time when 50 tables have already been produced, the cost of producing more tables is $20 per table. (So if this rate were constant, then the 51st table would cost exactly $20.)

(b) Note that when \( t = 1 \), the total number of tables manufactured is \( q = 100 \). So now by the chain rule we have

\[
\left. \frac{dC}{dt} \right|_{t=1} = \left( \frac{dC}{dq} \right|_{q=100} \cdot \left( \frac{dq}{dt} \right|_{t=1})
\]

\[
= \left((0.4q + 10)\right|_{q=100} \cdot (2t + 99)|_{t=1})
\]

\[
= (40 + 10) \cdot (2 + 99) = 50 \cdot 101 = 5050
\]

So at one hour after production begins, the total manufacturing cost is changing at a rate of 5050 \$/hour (or $5050 per hour).

2. Calculate \( \frac{d}{dx} \left( 4x^3 e^{\sin(2x)} \right) \). After computing the derivative, do not simplify your answer.

**Solution**

Use product rule. When differentiating the second factor, use chain rule twice.

\[
\frac{d}{dx} \left( 4x^3 e^{\sin(2x)} \right) = 4x^3 \cdot e^{\sin(2x)} \cdot \cos(2x) \cdot 2 + 12x^2 \cdot e^{\sin(2x)}
\]