Section 4.2: The Mean Value Theorem

**Thm:** (Mean Value Theorem, MVT)
Suppose $f$ is continuous on $[a,b]$ and $f$ is differentiable on $(a,b)$. Then there exists some number $c$ in $(a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

**Ex. 1**
Let $f(x) = x^{2/3}$. For each interval,
determine whether the hypotheses of the MVT are satisfied. If yes, find all values of \( c \) guaranteed to exist by the MVT.

(a) \([-1, 1]\]
(b) \([0, 8]\]

Solution:
For which intervals are the hypotheses of MVT satisfied for \( f(x) = x^{2/3} \)?

- **Where is \( f \) continuous?**
  - On its domain, which is \((-\infty, \infty)\)
- **Where is \( f \) differentiable?**

\[ y = x^{2/3} \]
f is differentiable on \((-\infty,0)\cup(0,\infty)\)

(a) (Interval \([-1,1]\))

Since \(f\) is not differentiable at \(x=0\) and \(0\) is in \((-1,1)\), hypotheses of MVT are not satisfied.

(b) (Interval \([0,8]\))

Yes, hypotheses are satisfied.

(Note: \(f\) is not diff. at \(x=0\), but \(0\) is not in \((0,8)\).)

So by MVT there is a number \(c\) in \((0,8)\) such that

Recall: \(f(x) = x^{2/3}\)

\[
f'(c) = \frac{f(8) - f(0)}{8 - 0}
\]

\[
\frac{2}{3} c^{-\frac{1}{3}} = \frac{8^{2/3} - 0^{2/3}}{8 - 0}
\]

\[
\frac{2}{3} c^{-\frac{1}{3}} = \frac{8^{2/3}}{8 - 0}
\]

\[
\frac{2}{3} c^{-\frac{1}{3}} = 8^{-\frac{1}{3}}
\]
Important Special Case of MVT

Thm: (Rolle's Theorem)

Suppose $f$ is continuous on $[a, b]$, that $f$ is differentiable on $(a, b)$, and $f(a) = f(b)$.

Then there exists a number $c$ in $(a, b)$ such that

$$f'(c) = 0$$