Section 3.4: Rectilinear Motion

**Basic Definitions**

\[ x(t) : \text{position along x-axis at time } t \]
\[ \rightarrow \text{or } s(t), \ y(t), \ h(t), \text{ etc.} \]
\[ v(t) : \text{velocity at time } t \]
\[ v(t) = \frac{dx}{dt} \]
\[ a(t) : \text{acceleration at time } t \]
\[ a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} \]

**Special Case: Constant Acceleration**

Freefall near surface of Earth is approximately constant.

If a particle has constant acceleration \( a \), then its position has the form

\[ x(t) = x_0 + v_0 t + \frac{1}{2} at^2 \]

“Galileo equation”

\( x_0 \) : initial position, \( x(0) \)
\( v_0: \) initial velocity, \( v(0) \)

\( a: \) (constant) acceleration

For gravity, \( a = -g = -9.8 \text{ m/s}^2 \) (Earth)

Q: Does \( x(t) \) above really give rise to constant acceleration?

A: \( x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \)

\( \rightarrow x(0) = x_0 + 0 + 0 = x_0 \)

\( x'(t) = v(t) = v_0 + at \)

\( \rightarrow v(0) = v_0 + 0 = v_0 \)

\( v'(t) = a(t) = a \) (constant!)

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**Ex. 1**

Stone is thrown upward at 20 m/s at a height of 100 m.

(a) What is max velocity of stone?

(b) What is max height?

(c) What is impact velocity?

(d) What is total distance traveled by
Stone until impact?

Use \( a = -g = -9.8 \text{ m/s}^2 \).

**Solution:**

Since the acceleration is constant, the height of stone has the form:

\[
h(t) = h_0 + v_0 t + \frac{1}{2} a t^2
\]

- \( h_0 = 100 \)
- \( v_0 = 20 \)
- \( a = -9.8 \)

\[
h(t) = 100 + 20t - 4.9t^2
\]

(a) The velocity is:

\[
v(t) = h'(t) = 20 - 9.8t
\]

Since \( v(t) \) is decreasing on \([0, \infty)\), the max velocity is \( v(0) = 20 \).

(b) To find max height, solve \( h'(t) = 0 \).
\[ h'(t) = 20 - 9.8t = 0 \quad \Rightarrow \quad t = \frac{20}{9.8} \]

Time when particle reaches maximum height.

Observe that \( h''(t) = -9.8 < 0 \) for all \( t \). So \( h \) is concave down on \((0, \infty)\). So \( t = \frac{20}{9.8} \) gives max height.

\[ h_{\text{max}} = h\left(\frac{20}{9.8}\right) = 120.4 \text{ m} \]

\[ h(t) = 100 + 20t - 4.9t^2 \]

(c) The impact velocity is the velocity of stone just as it hits the ground. First find when stone hits the ground.

\[ 0 = 100 + 20t - 4.9t^2 \]

Use quadratic formula.

\[ t = \frac{-2.926 \quad \text{or} \quad t = 6.998}{-2.926} \]

Time when stone hits ground.

So the impact velocity is
\[ V_{\text{impact}} = V(6.998) = 48.58 \] 

\[ V(t) = 20 - 9.8t \quad \text{Stone is falling down} \]

\[ \text{(d) \quad \text{Note: The total distance traveled is not simply } \Delta h = h(6.998) - h(0) = -100 \quad \text{and also not } |\Delta h| = 100.} \]

**Part 1** from \( t=0 \) to \( t=\frac{20}{9.8} \)

\[ d_1 = \int h(20/9.8) - h(0) \, ds = 20.4 \]

**Part 2** from \( t=20/9.8 \) to \( t=6.998 \)

\[ d_2 = \int h(6.998) - h(20/9.8) \, ds = |0 - 120.4| = 120.4 \]

\[ d_{\text{total}} = d_1 + d_2 = 140.8 \text{ m} \]
The position of a particle along x-axis is given by

\[ x(t) = 3t^3 - 40.5t^2 + 62t \]

for \( 0 \leq t \leq 8 \).

(a) When is particle at rest? \( v = 0 \)
(b) What is the total distance traveled by the particle? (from \( t=0 \) to \( t=8 \))

Solution:

(a) The velocity of particle is

\[ v(t) = 9t^2 - 81t + 162 \]

First solve \( v(t) = 0 \).

\[ 0 = 9(t-3)(t-6) \implies t = 3, \ t = 6 \]

Construct a sign chart for \( v(t) \) to solve both \( v(t) > 0 \) and \( v(t) < 0 \)
(b) Note: The total distance traveled is \[ \Delta x = x(8) - x(0) \] or \[ \Delta x | \].

The particle reverses direction at both \( t = 3 \) and \( t = 6 \). So we divide the path of the particle into three pieces.
\[ x(t) = 3t^3 - 40.5t^2 + 162t \]

**Part 1** \( t=0 \) to \( t=3 \)

\[ d_1 = (x(3) - x(0)) = |202.5 - 0| = 202.5 \]

**Part 2** \( t=3 \) to \( t=6 \)

\[ d_2 = (x(6) - x(3)) = |162 - 202.5| = 40.5 \]

**Part 3** \( t=6 \) to \( t=8 \)

\[ d_3 = (x(8) - x(6)) = |240 - 162| = 78 \]

\[ d_{\text{total}} = d_1 + d_2 + d_3 = 321 \]