1. For each part, write your answer on the provided line. You are not required to show work, but you may use the provided space for scratch work. For each part, **there is no partial credit.**

(a) Find the slope of the tangent line to the curve \( x^3 - y^3 = y - 1 \) at the point \((1, 1)\).

(b) The total revenue from selling \( x \) units of a certain product is \( R(x) = 40 - \frac{200}{x + 5} \). Using marginal analysis, estimate the revenue from selling the 6th unit.

(c) If \( x \) units are produced, then the total cost is \( C(x) = x^3 + 4x^2 + 60x + 200 \) and the selling price per unit is \( p(x) = 100 - 3x \). Find the level of production that maximizes the total profit.

(d) Use a linear approximation to estimate the value of \((16.32)^{1/4}\).

(e) Calculate the derivative of \( f(x) = x^x \). Your final answer must contain only \( x \).

(f) Find the equation of each horizontal asymptote of \( f(x) = \frac{2e^x - 5}{3e^x + 2} \). Write “NONE” as your answer if appropriate.

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2. Consider the function

\[ f(x) = e^{-x^2/2} \]

Find where \( f \) is concave down and find where \( f \) is concave up. Then find all inflection points (\( x \)- and \( y \)-coordinates). Write “NONE” for your answer if appropriate.

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3. Consider the function

\[ f(x) = \frac{1}{x^2 - 6x} \]

Find all vertical asymptotes of \( f \). Then find where \( f \) is decreasing and find where \( f \) is increasing. Finally determine the \( x \)-coordinates of all local extrema of \( f \) (and classify them as either a local minimum or a local maximum). Write “NONE” for your answer if appropriate.

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4. The surface area of a sphere is changing at a rate of \( 16\pi \) in\(^2\)/s when its radius is 3 in. At what rate is the volume of the sphere changing at that time?

*You must include correct units as part of your answer.*

**Hint:** If a sphere has radius \( R \), then its surface area \( A \) and volume \( V \) are given by

\[ A = 4\pi R^2, \quad V = \frac{4\pi}{3} R^3 \]

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5. A poster is to have a total area of 150 in\(^2\), which includes a central printed area, 1-inch margins at the bottom and sides, and a 2-inch margin at the top. What poster dimensions (in inches) will give the largest printed area? Use calculus to justify your answer.

*You must demonstrate that your answers really are the optimal dimensions.*
6. For each part, calculate the limit or show that it does not exist. If the limit is infinite, write “∞” or “−∞” as your answer, as appropriate.

(a) \[ \lim_{x \to 3^-} \left( \frac{x^2 + 6}{3 - x} \right) \]

(b) \[ \lim_{x \to 0} (1 - \sin(3x))^{1/x} \]

(c) \[ \lim_{x \to -3} \left( (x + 3) \tan \left( \frac{\pi x}{2} \right) \right) \]

7. A piece of cardboard that is 24 inches wide and 15 inches long is to be used to construct a box with an open top. To do this, congruent squares are cut from each corner of the cardboard, and the flaps are folded up and taped to form the sides of the box. What is the largest possible volume of such a box? Use calculus to justify your answer.

You must demonstrate that your answers really are the optimal dimensions.