1. For each part, write your answer on the provided line. You are not required to show work, but you may use the provided space for scratch work. For each part, \textit{there is no partial credit.}

(a) Calculate \( \lim_{x \to 0} \left( \frac{e^{2x} - 1}{\sin(5x)} \right) \) or determine that the limit does not exist.

(b) Find all (first-order) critical numbers of

\[ f(x) = x - \frac{3}{2}(x - 8)^{2/3} \]

If there are no critical numbers, write “NONE” as your answer.

(c) Use a linear approximation to estimate the value of \( \sqrt{35.9} \). Do not simplify your answer.

(d) The cost of producing \( x \) units is \( C(x) = 2x^2 + 5x + 8 \). Find the level of production (value of \( x \)) that minimizes the average cost. Recall that average cost is \( AC(x) = \frac{C(x)}{x} \).

(e) The cost of producing \( x \) units is \( C(x) = 3x^2 + 4x + 1000 \). Use marginal analysis to estimate the cost of producing the 41st unit.

(f) Find the equation(s) of all horizontal asymptotes of \( f(x) = \frac{4x^3 - 3x^2}{2x^3 + 9x + 1} \). If there are no horizontal asymptotes, write “NONE” as your answer.

2. Consider the function \( f \) and its derivatives below.

\[ f(x) = 2x + \frac{8}{x^2} , \quad f'(x) = \frac{2(x^3 - 8)}{x^3} , \quad f''(x) = \frac{48}{x^4} \]

Fill in the table below with information about the graph of \( y = f(x) \). For each part, write “NONE” as your answer if appropriate. (You may use the bottom or back of this page for scratch work.) \textit{You do not have to show work, and each part of the table will be graded with no partial credit.}

| vertical asymptote(s) | horizontal asymptote(s) | where \( f \) is decreasing | where \( f \) is increasing | \( x \)-coordinate(s) of local minima | \( x \)-coordinate(s) of local maxima | where \( f \) is concave down | where \( f \) is concave up | \( x \)-coordinate(s) of inflection point(s) |
3. A child flies a kite at a constant height of 30 feet and the wind is carrying the kite horizontally away from the child at a rate of 5 ft./sec. At what rate must the child let out the string when the kite is 50 feet away from the child?  
You must give correct units as part of your answer.

4. Find the absolute minimum and absolute maximum values of the function 
\[ f(x) = \frac{20x}{x^2 + 4} \]
on the interval \([-4, 0]\). You must show all work.

5. The parts of this problem are related!

(a) Show that \( \lim_{x \to \infty} \left( \frac{x}{x - 3} \right) = 1 \). You must show all work.

(b) Calculate the following limit or show it does not exist. You must show all work.
\[ \lim_{x \to \infty} \left( \frac{x}{x - 3} \right)^x \]
Hint: Use part (a) first to identify the appropriate indeterminate form.

6. According to postal regulations, the sum of the girth and length of a parcel may not exceed 90 inches. What are the dimensions (in inches) of the parcel with the largest possible volume that can be sent, if the parcel is a rectangular box with two square sides?  
You must demonstrate that your answer really does give the largest volume.

7. This problem asks about the Mean Value Theorem (MVT).

(a) Explain precisely why the function \( f(x) = |3x - 6| \) does not satisfy the hypotheses of the MVT on the interval \([-1, 4]\).

(b) The function \( f(x) = \sqrt{x} \) satisfies the hypotheses of the MVT on the interval \([0, 36]\). Find all values of \( c \) guaranteed to exist by the MVT.