1. For each limit, calculate the value or show that it does not exist. If the limit is $+\infty$ or $-\infty$, that should be your answer instead of “does not exist”. Show all work.

(a) \[ \lim_{x \to 0} \left( \frac{1 - \cos(9x)}{x^2} \right) \] 7 pts

(b) \[ \lim_{x \to 0} (1 - 3x)^{5/x} \] 7 pts

2. Find an equation of the line tangent to the curve

\[ \sin \left( \frac{\pi x}{y} \right) = x - 8y \]

at the point (8, 1). 12 pts

3. A person 5 feet tall stands stationary 8 feet from the point $P$, which is directly beneath a lantern that falls toward the ground. At the moment when the lantern is 15 feet above the ground, the lantern is falling at a speed of 4 feet per second. At what rate is the length of the person’s shadow changing at this moment?

You must give correct units as part of your answer. 14 pts

4. The concentration of a certain drug in the bloodstream $t$ hours after the drug is injected is modeled by the following formula.

\[ C(t) = \frac{100t}{t^2 + 1} \]

(The concentration is measured in micrograms per milliliter.) Use a linear approximation to estimate the change in the concentration over the time period from 2 to 2.1 hours after injection. Also indicate whether the concentration increases or decreases. 10 pts

5. Consider the function $f$ and its derivatives below.

\[ f(x) = \frac{2x^3 + 3x^2 - 1}{x^3}, \quad f'(x) = \frac{3 - 3x^2}{x^4}, \quad f''(x) = \frac{6x^2 - 12}{x^5} \]

Intervals should be given in a comma-separated list and should be as inclusive as possible. For each part, write “does not exist” as your answer if appropriate. You must show all work.

(a) Find all horizontal asymptotes of $f$. 4 pts

(b) Find all vertical asymptotes of $f$. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of $f$. 3 pts

(c) Find where $f$ is decreasing and find where $f$ is increasing. Then calculate the $x$-coordinates of all points of local extrema. 7 pts
(d) Find where \( f \) is concave down and find where \( f \) is concave up. Then calculate the \( x \)-coordinates of all points of inflection.

6. Let \( f(x) = 4(x - 3)^{1/3} - \frac{1}{3}x + 1 \). Note: The domain of \( f \) is \((-\infty, \infty)\).

(a) Calculate all critical numbers of \( f \). For each number you find, you must clearly indicate in your work why it is a critical number.

(b) What are the global extreme values of \( f \) on the interval \([2, 30]\)?

7. Find the maximum possible area of a rectangle inscribed in the region between the graph of \( f(x) = e^{-x^2/12} \) and the \( x \)-axis. Note: The graph of \( y = f(x) \) has no \( x \)-intercepts.

\[ y = e^{-x^2/12} \]

You must clearly demonstrate that your answer really is the maximum area!