Book #1 of 2

Name: ____________________________________________

ID# (last 4 digits): ____________________________ Section: ____________________________

Unless stated otherwise, you must show all work clearly using proper notation and explain
your reasoning in English where appropriate. Answers must be justified using techniques that
have been taught in this course, and answers without such justification may receive less than
full credit – or no credit at all – even if the answer is correct.

This exam is closed book. Calculators, electronic devices, notes, books, formula sheets, and
other outside materials are not allowed. Phones must be turned off and put away.

Unless otherwise stated, give exact answers: e.g., write $\pi$ and $\sqrt{2}$ instead of 3.14 and 1.41.
However, when an expression simplifies to a well-known value, you must use that value. For
example, you must write 1 instead of $e^0$, and you must write $\frac{1}{2}$ instead of $\cos\left(\frac{\pi}{3}\right)$.

You must justify all uses of L’Hospital’s Rule (LR). If you use LR for any calculation, you
must indicate why LR is applicable. It is also preferred, but not necessary, that you use the
symbol “$\frac{d}{dx}$” instead of a normal equals sign to indicate the exact step in which you use LR.

This exam has 7 questions, printed in 1 booklet(s), for a total of 100 points.

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1. For each limit, calculate the value or show that it does not exist. If the limit is $+\infty$ or $-\infty$, that should be your answer instead of “does not exist”. Show all work.

(a) $\lim_{x \to 0} \left( \frac{1 - \cos(9x)}{x^2} \right)$

value of limit: _________________

(b) $\lim_{x \to 0} (1 - 3x)^{5/x}$

value of limit: _________________
2. Find an equation of the line tangent to the curve

\[ \sin \left( \frac{\pi x}{y} \right) = x - 8y \]

at the point (8, 1).

equation of line: _____________________
3. A person 5 feet tall stands stationary 8 feet from the point $P$, which is directly beneath a lantern that falls toward the ground. At the moment when the lantern is 15 feet above the ground, the lantern is falling at a speed of 4 feet per second. At what rate is the length of the person’s shadow changing at this moment?

*You must give correct units as part of your answer.*

rate of change of length of shadow: ______________
4. The concentration of a certain drug in the bloodstream \( t \) hours after the drug is injected is modeled by the following formula.

\[ C(t) = \frac{100t}{t^2 + 1} \]

(The concentration is measured in micrograms per milliliter.) Use a linear approximation to estimate the change in the concentration over the time period from 2 to 2.1 hours after injection. Also indicate whether the concentration increases or decreases.

change in concentration: ______________________

increase or decrease? ______________________
Book #2 of 2

Name: ____________________________

ID# (last 4 digits): __________________ Section: _______________________

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You must justify all uses of L'Hospital's Rule (LR). If you use LR for any calculation, you must indicate why LR is applicable. It is also preferred, but not necessary, that you use the symbol \( \frac{d}{dx} \) instead of a normal equals sign to indicate the exact step in which you use LR.

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5. Note: This problem continues onto the next page.

Consider the function $f$ and its derivatives below.

$$f(x) = \frac{2x^3 + 3x^2 - 1}{x^3}, \quad f'(x) = \frac{3 - 3x^2}{x^4}, \quad f''(x) = \frac{6x^2 - 12}{x^5}$$

Intervals should be given in a comma-separated list and should be as inclusive as possible. For each part, write “does not exist” as your answer if appropriate. You must show all work.

(a) Find all horizontal asymptotes of $f$.

horizontal asymptote(s): ________________

(b) Find all vertical asymptotes of $f$. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of $f$.

vertical asymptote(s): ________________
Note: This is a continuation of the problem on the previous page.

\[ f(x) = \frac{2x^3 + 3x^2 - 1}{x^3} \]  \[ f'(x) = \frac{3 - 3x^2}{x^4} \]  \[ f''(x) = \frac{6x^2 - 12}{x^5} \]

**Intervals should be given in a comma-separated list and should be as inclusive as possible. For each part, write “does not exist” as your answer if appropriate. You must show all work.**

7 pts

(c) Find where \( f \) is decreasing and find where \( f \) is increasing. Then calculate the \( x \)-coordinates of all points of local extrema.

decreasing: ____________________________

increasing: ____________________________

\( x \)-coordinate(s) of local minima: ____________________________

\( x \)-coordinate(s) of local maxima: ____________________________
Note: This is a continuation of the problem on the previous page.

\[ f(x) = \frac{2x^3 + 3x^2 - 1}{x^3} \quad , \quad f'(x) = \frac{3 - 3x^2}{x^4} \quad , \quad f''(x) = \frac{6x^2 - 12}{x^5} \]

Intervals should be given in a comma-separated list and should be as inclusive as possible. For each part, write “does not exist” as your answer if appropriate. You must show all work.

7 pts

(d) Find where \( f \) is concave down and find where \( f \) is concave up. Then calculate the \( x \)-coordinates of all points of inflection.

concave down: __________________________

concave up: __________________________

\( x \)-coordinate(s) of inflection point(s): __________________________
6. Let \( f(x) = 4(x - 3)^{1/3} - \frac{1}{3}x + 1 \). Note: The domain of \( f \) is \((−\infty, \infty)\).

(a) Calculate all critical numbers of \( f \). For each number you find, you must clearly indicate in your work why it is a critical number.

Critical number(s): 

(b) What are the global extreme values of \( f \) on the interval \([2, 30]\)?

Global minimum value: 

Global maximum value: 
7. Find the maximum possible area of a rectangle inscribed in the region between the graph of \( f(x) = e^{-x^2/12} \) and the \( x \)-axis. Note: The graph of \( y = f(x) \) has no \( x \)-intercepts.

You must clearly demonstrate that your answer really is the maximum area!

maximum area: ________________
This page is for scratch work. Do not detach this sheet.
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