Book #1 of 2

Name: ________________________________

ID# (last 4 digits): ____________________  Section: ________________________________

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your reasoning in English where appropriate. Answers must be justified using techniques that
have been taught in this course, and answers without such justification may receive less than
full credit – or no credit at all – even if the answer is correct.

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However, when an expression simplifies to a well-known value, you must use that value. For
example, you must write 1 instead of $e^0$, and you must write $\frac{1}{2}$ instead of $\cos\left(\frac{\pi}{3}\right)$.

This exam has 7 questions, printed in 1 booklet(s), for a total of 100 points.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>21</td>
<td></td>
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<td>2.</td>
<td>20</td>
<td></td>
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<td>3.</td>
<td>9</td>
<td></td>
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<tr>
<td>Total:</td>
<td>100</td>
<td></td>
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</tbody>
</table>
1. Note: This problem continues onto the next page.

A scientist measures the temperature $T$ (measured in kelvins) of a certain metal bar $t$ seconds after the measurements have begun. The following equation models the data observed by the scientist.

$$T(t) = \frac{30}{e^t} + 10e^t$$

All final answers must be integers or simplified fractions with proper units.

(a) Calculate $T'(\ln(3))$ and explain its meaning in the context of this problem.

$$T'(\ln(3)) = \text{_______________________}$$

(b) Describe in plain English, as precisely and specifically as you can, what the quantity $Q = T(273) - T(152)$ represents in the context of this problem.
The scientist also observes that the length $L$ of the metal bar (measured in centimeters) depends on its temperature through the following equation.

$$L(T) = 0.01T^3 + \frac{64,000}{T}$$

**All final answers must be integers or simplified fractions with proper units.**

(c) Calculate $L'(40)$ and explain its meaning in the context of this problem.

$$L'(40) = \phantom{0}$$

(d) At what rate is the length of the bar changing with respect to time exactly $\ln(3)$ seconds after the measurements begin?

rate of change: \phantom{0}
2. Note: This problem continues onto the next page.

For each limit, calculate the value or show that it does not exist. Show all work.

(a) \[ \lim_{u \to 4} \left( \frac{(u + 6)^2 - 25u}{u - 4} \right) \]

value of limit: ________________

(b) \[ \lim_{s \to 1} g(s) \] where \( g \) is the function \( g(s) = \begin{cases} \sqrt{1 - s} & , \ s \leq 1 \\ \frac{s^2 - s}{s - 1} & , \ s > 1 \end{cases} \)

value of limit: ________________
Note: This is a continuation of the problem on the previous page.

For each limit, calculate the value or show that it does not exist. Show all work.

(c) \[ \lim_{h \to 0} \left( \frac{\sin(7 + h) - \sin(7)}{h} \right) \]

*Hint:* Recall the limit definition of the derivative!

value of limit: _______________

(d) \[ \lim_{x \to 6} \left( \frac{\frac{1}{36} - x^{-2}}{x^2 - 36} \right) \]

value of limit: _______________
3. The position of a particle along the $x$-axis at time $t$ is given by

$$x(t) = t^3 - 6t^2 + 9t + 10$$

(a) When is the particle retreating? When is the particle advancing?

*Write your answer using interval notation.*

retreating: __________________

advancing: __________________

(b) What is the total distance traveled by the particle during the period $0 \leq t \leq 7$? *(The table below shows selected values of the position.)*

*Write your answer as an integer or simplified fraction.*

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(t)$</td>
<td>10</td>
<td>14</td>
<td>12</td>
<td>10</td>
<td>14</td>
<td>30</td>
<td>64</td>
<td>122</td>
</tr>
</tbody>
</table>

distance traveled: ________________
This page is for scratch work. Do not detach this sheet.
This page is for scratch work. Do not detach this sheet.
Book #2 of 2

Name: ________________________________

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4. Find the values of $a$ and $b$ which would make the following function continuous at $x = 0$. 

$$f(x) = \begin{cases} 
\frac{4 - \sqrt{16 + 49x^2}}{ax^2}, & x < 0 \\
-23, & x = 0 \\
\frac{\tan(2bx)}{x}, & x > 0 
\end{cases}$$

Write “does not exist” for your answer if appropriate. Otherwise, your answers should be integers or simplified fractions. You must use calculus to give a full, clear justification for your answer.

value of $a$: ______________ 

value of $b$: ______________
5. Let \( f(x) = \frac{1}{3}x^3 \) and let \( g(x) = x^2 + 15x - 3 \). Find all values of \( a \) for which the tangent lines to \( y = f(x) \) and \( y = g(x) \) at \( x = a \) are parallel.

value(s) of \( a \): ______________________

6. Calculate \( f'(x) \) where \( f \) is the function below.

\[
f(x) = \left( \frac{x^8 \sin(3x)}{\ln(x) - \ln(11)} \right)^{2/3}
\]

After calculating the derivative, do not simplify your answer.
7. Suppose $f$ and $g$ are differentiable for all $x$. For each part, use the table below or explain why there is not enough information.

<table>
<thead>
<tr>
<th></th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-1</td>
<td>-4</td>
<td>4</td>
<td>2</td>
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<td>-1</td>
<td>-3</td>
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<td>2</td>
<td>-4</td>
<td>3</td>
<td>1</td>
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(a) Let $F(x) = \frac{f(x)}{g(x)}$. Calculate $F'(0)$.

$$F'(0) = \text{________________}_$$

(b) Let $G(x) = f\left(xg(x)\right)$. Calculate $G'(1)$.

$$G'(1) = \text{________________}_$$
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