Oral Qualifying Exam Syllabus
for James Holland

Exam Date: 2019-04-30
Exam Committee: Eric Allender,
Gregory Cherlin,
Grigor Sargsyan (chair), and
Simon Thomas.

1. Set Theory
   i. Basic Set Theory
      • König’s theorem;
      • generalized $\Delta$-system lemma;
      • Shoenfield’s absoluteness theorem;
      • the constructible hierarchy, $L \not\models \diamond + \text{GCH}$;
      • Suslin’s problem: $\diamond$ denies $\text{SH}$, and $\text{MA} + \neg \text{CH}$ implies $\text{SH}$;
      • existence of an $\aleph_1$-Aronszajn tree.
   ii. Advanced Set Theory
      • Forcing theorems, chain conditions, closure conditions;
      • forcing $\text{CH}$, $\neg \text{CH}$, and $\diamond$;
      • product forcing;
      • iterated forcing, consistency of $\text{ZFC + MA + CH}$;
      • Cohen forcing;
      • Easton forcing;
      • measurable cardinals, elementary embeddings, and ultrapowers.

2. Model Theory
   • Completeness, compactness, Löwenheim–Skolem, Łoś–Vaught, Tarski–Vaught theorems and tests;
   • quantifier elimination;
   • omitting types theorem;
   • atomic, and prime models;
   • saturated models;
   • homogeneous models;
   • indiscernibles;
   • $\omega$-stable theories;
   • $\aleph_0$-categoricity, Morley’s categoricity theorem;
   • Morley rank, transcendentals.

3. Recursion Theory
   • Primitive recursive, and recursive functions;
   • recursively enumerable sets;
   • the recursion theorem;
   • relative computability;
   • the arithmetic hierarchy;
   • Turing degrees;
   • simple, and complete sets.