MATH 338: Homework #2

Due: Thursday, February 14, 2019

Solve the below problems concerning basic epidemiology and population genetics.

1. Consider the transition probabilities for the SIS model as seen in class. Analogously the computation (in class) of $P(I(t+\Delta t) = i+1 \mid I(t) = i)$, find the following transition probabilities:
   (a) $P(S(t+\Delta t) = s+1 \mid S(t) = s)$
   (b) $P(S(t+\Delta t) = s \mid S(t) = s)$

2. In class, we showed that for a single locus with two alleles (A and a) in a monecious diploid species, we showed that

$$P(\text{offspring is } AA) = \left(f_{AA} + \frac{f_{Aa}}{2}\right)^2$$
$$P(\text{offspring is } Aa) = 2\left(f_{AA} + \frac{f_{Aa}}{2}\right)\left(f_{aa} + \frac{f_{Aa}}{2}\right)$$
$$P(\text{offspring is } aa) = \left(f_{aa} + \frac{f_{Aa}}{2}\right)^2.$$

   Show *explicitly* that the right-hand sides of the above sum to one (i.e. without using the fact that the three events partition the probability space).

3. Locus ℓ admits 4 different alleles. Locus k, on a different chromosome, admits 2 alleles. The organisms is diploid.
   (a) What are the number of possible combined genotypes at these two loci?
   (b) What is the number of genotypes of the (haploid) gametes of this organism?

4. Let ℓ be a locus on the X chromosome of humans. The gene has three alleles (A, a, and $\bar{a}$). Consider a second locus on an autosomal chromosome admitting alleles B and b. Let the population of males and females have genotypes as follows:

$$\text{Males: } ABB, ABB, aBb, \bar{a}Bb, \bar{a}bb$$
$$\text{Females: } AAbb, aaBb, a\bar{a}bb, \bar{a}ABB, \bar{a}\bar{a}BB, aABb, a\bar{a}BB$$

Find the allele frequencies f_A and f_B for the population (males and females).

5. Exercise 3.1.3 in Chapter 3 of the online notes (page 11).

6. Recall that

$$f_{AA} + f_{Aa} + f_{aa} = 1.$$

Prove that the genotype frequencies (again, recall that this is for the simple case of one locus, 2 alleles, monecious diploid species) are in Hardy-Weinberg equilibrium if and only if

$$f_{Aa}^2 = 4 f_{AA} f_{aa}.$$

7. You are studying a hypothetical species of butterfly. It has one gene that controls wing color, with two alleles B and Y. Genotype BB butterflies have blue wings, genotype YY have yellow wings, and genotype BY have green wings. You sample butterflies in a population of mixed colors and find that the frequencies of blue, yellow, and green butterflies $0.2, 0.3$, and 0.5, respectively. Is the population in Hardy-Weinberg equilibrium? If not, what would the Hardy-Weinberg equilibrium be given the actual allele frequencies?