Exam 1 Review Problems

For details on exam coverage and a list of topics, please see the class website. Note that this is a set of review problems, and NOT a practice exam.

1. Three sets of base pairs in DNA (such a triplet is called a *codon*) code for an amino acid. Assuming no other information, how many possible amino acids should exist? How many actually exist? Discuss.

2. Consider a monecious species with non-overlapping generations and a locus with two alleles A and a. In its home territory, the population is in permanent Hardy-Weinberg equilibrium and the frequency of A is $2/3$. A colony is established nearby and the initial (generation $t = 0$) frequency of A in the colony is $1/6$. With probability $3/4$ each child in the colony is the offspring of a random mating of a parent between two parents in the colony, and with probability $1/4$ is the offspring of a random mating of a parent in the colony and a parent from the home territory.

   Assume infinite populations.

   Derive and solve a difference equation for $f_A(t)$, the frequency of A in the colony in generation t. (Show reasoning.) Find $\lim_{t\to\infty} f_A(t)$ and comment.

3. A population consists of two sub-populations I and II. They are infinite and generations do not overlap. Consider a locus with two alleles A and a. Let $f_A^I(t)$ and $f_A^{II}(t)$ be the frequencies of allele A in sub-populations I and II, respectively.

   Individuals in each new generation of population I are produced by randomly selecting one parent from population I and one from population II. Individuals in each new generation of population II are produced by randomly selecting one parent from generation I; the second parent is randomly selected from population I with probability 0.4, and randomly selected from population II with probability 0.6.

   (a) $f_A^I(t)$ and $f_A^{II}(t)$ will satisfy a system of equations of the form

   $$f_A^I(t+1) = \alpha f_A^I(t) + \beta f_A^{II}(t)$$
   $$f_A^{II}(t+1) = \delta f_A^I(t) + \gamma f_A^{II}(t)$$

   What are the constants α, β, δ, and γ? (*Hint:* Think in terms of the percentages that populations I and II contribute respectively to the allele pools of the next generation.)

   (b) It can be shown that

   $$c := f_A^I(t) + \frac{5}{7} f_A^{II}(t)$$

   is constant in time t. Assume $c = 3/7$ and $f_A^{II}(0) = 1/5$. Find a different equation for $f_A^{II}(t)$ alone, solve it, and determine $\lim_{t\to\infty} f_A^{II}(t)$.

4. Consider the one locus, two allele selection model. Assume that the genotype aa is lethal and that the population has an equilibrium frequency for a of 0.40. If the fitness of Aa is 1, what is the fitness of the AA genotype?

5. In the plant species *Grandi floria*, most individuals have large flowers. However, plants that are homozygous recessive at the G allele have small flowers. If there are 75 plants with large flowers, and 25 plants with small flowers, calculate the following, **assuming Hardy-Weinberg equilibrium**:

   (a) The frequency of alleles G and g.

(b) The genotype frequencies.

6. Problem 3.2.8 in the online notes (page 18). Note that this deals with a locus of a **dioecious** species on an autosomal chromosome. The main result is that it takes **two** generations, instead of one, to reach Hardy-Weinberg equilibrium.

7. Three possible alleles (S, T, U) can appear at a locus ℓ of a monecious species. In an experiment, a population (generation $t = 0$) is prepared with genotype frequencies

$$\begin{aligned} f_{SS}(0) &= 0.2 \quad f_{ST}(0) &&= 0.1 \\ f_{SU}(0) &= 0.1 \quad f_{TT}(0) &&= 0.3 \\ f_{TU}(0) &= 0.2 \quad f_{UU}(0) &&= 0.1 \end{aligned}$$

(a) What are the allele frequencies $f_S(0)$, $f_T(0)$, and $f_U(0)$ of generation 0?

(b) Assume that the next generation ($t = 1$) is effectively infinite produced by random mating. What will the frequency of genotype TU and the frequency of allele T be in generation $t = 1$? Explain where and how the infinite population and random mating assumptions are used in the calculation of your answer.

(c) How many generations will it take for all genotype frequencies to reach equilibrium values?

(d) Suppose instead that S mutates to T in the course of reproduction with probability 0.1 and U mutates to T with probability 0.2, but no other mutations take place. What will be the frequency of allele T in the first generation in this case?

8. In class, it was claimed that a third fixed point of the selection model was given by

$$\bar{p} = \frac{w_{Aa} - w_{aa}}{2w_{Aa} - w_{AA} - w_{aa}}.$$

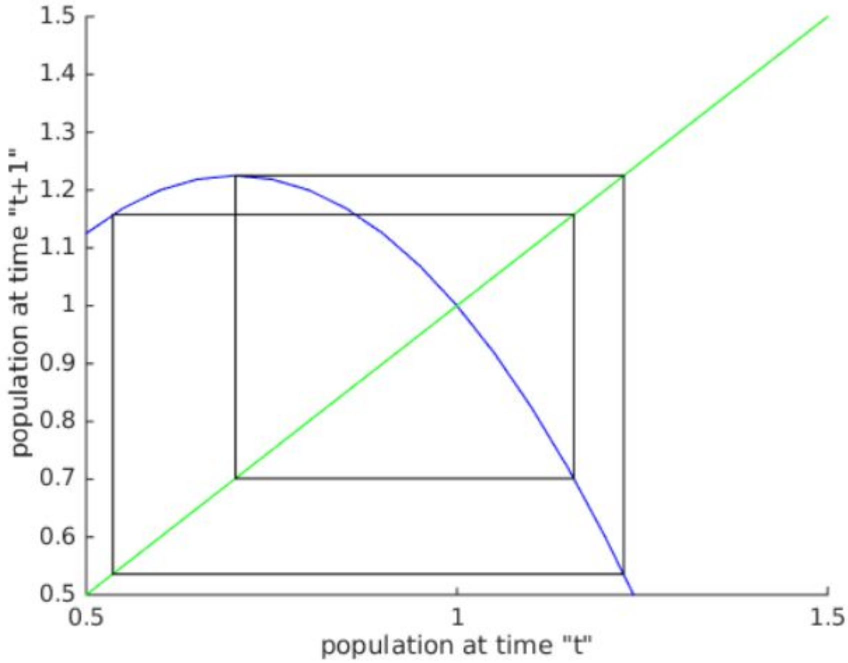
Derive this formula, and show that $\bar{p} \in (0, 1)$ if and only if either (i) $w_{Aa} > w_{AA}$ and $w_{Aa} > w_{aa}$, or (ii) $w_{Aa} < w_{AA}$ and $w_{Aa} < w_{aa}$.

9. Find a solution $x(t)$ to

$$\begin{aligned} x(t+1) &= 2x(t) + 8x(t-1) \\ x(0) &= 0 \\ x(1) &= 3 \end{aligned}$$

Describe the behavior of $x(t)$ for large t.

10. Below is a cobweb diagram for a certain difference equation $P_{t+1} = f(P_t)$:



Suppose that P_0 is (approximately) 1.22. Sketch below the plot of P_t for t from 0 to 10.