

MATH 495: Mathematics of Cancer

Quiz 6

NAME: _____

Date: April 17, 2018

Answer the following questions on this sheet of paper. No calculators or other electronic devices are permitted.

Consider the (non-dimensionalized) tumor-immune model presented in class:

$$\begin{aligned} \dot{x} &= \sigma + \rho \frac{xy}{\eta + y} - \mu xy - \delta x \\ \dot{y} &= \alpha y(1 - \beta y) - xy \end{aligned} \tag{1}$$

In the following, we will consider modifications to system (1) which take into account different biological observations.

1. Suppose that the tumor is growing **exponentially**. Modify system (1) to take this into account. That is, rewrite the ODEs for $x(t)$ and $y(t)$ modeling tumor exponential growth.
Hint: you will have to replace one term.

2. In addition to the assumption in Problem 1, assume further that the tumor does not inhibit the signaling of the immune (effector) cells. Modify your equations from Problem 1 to also include this phenomenon.

3. Considering the system from Problem 2, how many **biologically significant** steady states does your system possess? Note that your answer may depend on relations between parameters. Provide justification.
Hint: I am **not** asking you to find formulas (but you can if you want). I suggest drawing nullclines.