MATH 495: Mathematics of Cancer

Quiz 4

NAME: ______________________________ Date: March 8, 2018

Answer the following question on this sheet of paper. No calculators or other electronic devices are permitted.

Consider the basic model of the cell-cycle describing proliferating and quiescent (really senescent) cells, where transitions to proliferation are prohibited, **but quiescent cells undergo apoptosis (i.e. death) at a constant rate $\boldsymbol{\mu_q}$**:

$$\begin{aligned}\dot{P} &= (\beta - \mu_p - r_0(N))\,P,\\ \dot{Q} &= r_0(N)P - \mu_q Q.\end{aligned} \tag{1}$$

As usual, all parameters are positive, and $r_0(N)$ is an increasing function of the total cellular population N.

(a) Transform the system (1) to the PN plane, i.e. to a closed system of equations describing the proliferating and total cell populations only.

(b) Now assume that $\beta - \mu_p < \ell_0$, where ℓ_0 is the (well-defined) limit of $r_0(N)$. Show that your system in part (a) has a **unique steady state with positive components**, say $(\bar{P}, \bar{N})$. *Hint:* How many solutions does $\dot{P} = 0$ have?

(c) Show that the steady state $(\bar{P}, \bar{N})$ found in part (b) is locally stable. Note that you may not have an exact formula for $(\bar{P}, \bar{N})$, but I claim you can still make an argument based on the equations it satisfies.