

MATH 421: Advanced Calculus for Engineering
Review Quiz

NAME: _____

Due: January 28, 2016

This quiz is designed to give you a basic review of some concepts from CALC 1-4 that will be important for this course. It will also be useful for me to gauge the general level of the class. Be aware that the set is not exhaustive, and that you will be required to know other material from previous math courses not appearing here. Please review any concepts that are difficult and/or unfamiliar; I will also be happy to meet and discuss any questions you may have. Show work where applicable; solutions submitted with no work will not earn any credit.

1. What is the largest value the function $f(x) = \frac{2x}{e^x}$ can take when $x \geq 0$? At what value of x does this occur?

2. Evaluate $\int_0^1 x \ln x \, dx$.

3. Suppose $z = \frac{x}{y}$ and $x = se^t$ and $y = 1 + se^{-t}$. That is, z is a function of x and y , which are in turn functions of s and t (i.e. z is really a function of s and t). Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. Note that your answer should have **ONLY** s 's and t 's (and no x and y terms).

4. Find a partial fraction expansion of the following function:

$$f(x) = \frac{2x^2 - x + 4}{x^3 + 4x}$$

5. Find the general solution of the following first-order linear ordinary differential equation (ODE)

$$\cos^2 x \sin x \frac{dy}{dx} + \cos^3(x)y = 1.$$

Find the largest interval which contains $x = \frac{\pi}{4}$ on which the solution exists.

6. Solve the following second-order linear ODEs:

(a) $2y'' - 5y' - 3y = 0$

(b) $y'' - 10y' + 25y = 0$

(c) $y'' + 4y' + 7y = 0, \quad y(0) = 1, y'(0) = 2.$

Note that (c) is an initial-value problem, while in (a) and (b) you should find the general solution.