Exam 3 Information

Date: May 10th, 2016

Location: Normal classroom (TIL-258, Livingston Campus)

Time: 8:00-11:00 pm

Notes:

1. Calculators, cell phones, computers, etc. are **prohibited**. All calculations will be able to be completed by hand.
2. You will provided a sheet of standard Laplace transforms, which you can use freely (except if you are asked to compute a Laplace transform from the **definition**). The Laplace transform table will be exactly as provided for Exam 1, and can be currently found on the course website.
3. No other formula sheets will be provided. The exam is closed book/closed notes, and you will be expected to memorize the solutions of certain PDE boundary-value problems.
4. The exam will be cumulative, but will be slightly more focused on the material since Exam 2 (Chapter 13 PDE material).
5. There will be **at least** one question taken verbatim from one of the previous two exams.
6. I have designed the exam to take approximately two hours, but you may stay the full three if you'd prefer.
7. I can be in my office on Monday, May 9th. I have a meeting in the early/mid afternoon, but will be in my office sometime by the afternoon/early evening. I can also be in my office on Tuesday before the exam, and am available to meet at other times via appointment. Please do not hesitate to email me to set something up.

Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions.
3. Solve all problems from Exam 1 and Exam 2. Remember, there will be at least one problem from one of these two exams.
4. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material covered in the course is fair-game for the exam. Regarding material from the first two exams, I suggest looking at their respective information sheets to get an extensive list, although I will highlight some important material below. As usual, be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered.

Laplace transforms

(a) Definition of Laplace and Inverse Laplace Transforms. In particular properties and use in relation to solving first and second order ODEs.

(b) Be very comfortable with the Laplace table. You should be able to quickly identify the correct entry to use for each Laplace/inverse Laplace question.

(c) Laplace Transform of an integrals and convolutions. Be able to solve integral equations.

(d) Laplace Transform of Heaviside step and delta functions, and their uses in initial value ODE problems.

Linear Algebra

(a) Solving systems of linear equations, using augmented matrix form. Know about the number of solutions, free variables (parameters), etc. **This is very important, and used in most of this chapter, so really have it down.** Going from augmented matrix to a parameterized set of solutions. Equivalence of systems of linear equations and matrix vector equation ($A\vec{x} = \vec{b}$). Also, reduced row-echelon form.

(b) Determinants and their properties and their relation to matrix inverses (as well as computation of matrix inverse).

(c) Eigenvalues and eigenvectors. In particular, their computation and relation. Also, properties of eigenvectors of a symmetric matrix, and their relation to the diagonalization of a matrix.

Fourier Series

(a) Inner products of functions, and extension of the Linear Algebra terminology to function spaces. For instance, orthogonality, norm, linear combination, etc.

(b) Fourier series. Know the definition, the orthogonal basis being used, and the interval where the expansion holds. **Have the formula memorized.**

(c) Convergence of the Fourier series to the actual function $f(x)$. For what x do the two coincide (Answer: points of continuity)? When they don't coincide, what is the value of the Fourier series at the point x (Answer: $\frac{f(x-)+f(x+)}{2}$)?. Know how to represent both graphically.

(d) Even and odd functions, and their corresponding Fourier series as cosine and sine series, respectively. Extensions to Fourier series on $[0, L]$ (as opposed to $(-p, p)$). That is, half-range expansions for even, odd, and identity reflections.

PDEs and Boundary Value Problems

(a) Boundary value problems (BVPs) for ODEs. Definition of an eigenvalue and corresponding eigenfunction of an ODE with parameter λ. How boundary conditions restrict the possible values of λ where non-trivial solutions exist. You should be familiar with second order, constant coefficient, homogeneous ODEs for this material.

(b) Basics of PDEs. How problems are formulated (usually initial and boundary conditions). Linear vs. nonlinear, principle of superposition, and classification of second order, constant coefficient, linear PDEs (hyperbolic, parabolic, and elliptic).

(c) Method of separation of variables. What it says, and how it (often) reduces a PDE to a system of ODEs. Furthermore, the method of solving these ODEs (in terms of the separation constant λ), and combining them to form a solution of the original PDE.

(d) Heat equation formulation. Understand where the equation comes from, and what it represents. Formulation as a boundary value problem with initial conditions. Especially the different types of boundary conditions (Dirichlet, Neumann, mixed), and what they represent physically. In particular fixed temperature vs. insulated.

(e) Similar to (d) with the wave equation. Know the basic idea of the derivation, and what the standard boundary conditions represent, so that you can formulate a BVP from words.

(f) Solutions of both the **standard** heat and wave equations. You should have the formulas memorized for the solution of the following heat equation BVP:

$$\begin{aligned} \frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0 \\ u(0,t) &= 0, \qquad u(L,t) = 0, \quad t > 0 \\ u(x,0) &= f(x), \quad 0 < x < L \end{aligned}$$

and the following wave equation BVP:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= a^2\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0 \\ u(0,t) &= 0, \qquad u(L,t) = 0, \quad t > 0 \\ u(x,0) &= f(x), \qquad \frac{\partial u}{\partial t}(x,0) = g(x), \quad 0 < x < L \end{aligned}$$

You should certainly understand the derivation of both formulas (via separation of variables; you will have to do something similar), but have the formulas **memorized** for both of these BVPs, so that you can compute $u(x,t)$ for specific initial conditions $f(x)$ and $g(x)$ (and possibly a specified length L).

(g) Laplace's equation. This is likely the situation where you will have to solve, **from scratch**, a BVP, starting with separation of variables, and incorporating boundary conditions, etc. Know the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

and the entire procedure from beginning to end (including the argument for the possible eigenvalues). You will have to match at least one set of non-homogeneous boundary conditions (via a Fourier series argument, i.e. to recognize the coefficients as the coefficients in some Fourier series, and use the formulas from Chapter 12), and write down the formula for the full solution as an infinite series. Most domains will be rectangular in the xy-plane, but you should also know the semi-infinite example discussed on Monday's class (5/2).