Exam 2 Information

Date: April 18th, 2016

Location: Normal classroom (TIL-258, Livingston Campus)

Time: In-class (8:40-10:00 am)

Notes:

1. Calculators are **prohibited**. All calculations will be able to be completed by hand.
2. I can be in my office on Friday, if you'd like to meet in preparation for the exam (Hill 216). Please just email me and let me know if you'd like to meet. I will also respond to emails during the weekend, but unfortunately I *cannot* be in my office on Saturday or Sunday.
3. There will **not** be a formula sheet for this exam. Laplace transforms are not covered, and you are expected to memorize certain important formulas (Fourier series, heat equation, etc.)

Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions.
3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material up to and including what was covered on Monday's (April 11th) course is fair-game for the exam. Roughly speaking, this is the end of the Linear Algebra chapter (Sections 8.10 and 8.12), all of the (covered) Fourier series chapter, and the beginning of the PDE material. You should also be familiar with ODE boundary value problems (Section 3.9), as well as the vector space terminology from Section 7.6 (this is used when talking about inner products, linear independence, etc. for function spaces in Chapter 12). Some key topics to review are given below. But be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered.

Linear Algebra

(a) Symmetric and orthogonal matrices. Know all covered properties and equivalent definitions of both. In particular, relations between the eigenvalues and eigenvectors, and how to construct an orthogonal matrix from a symmetric one.

(b) Matrix diagonalization. Be able to show when (and when not) a given matrix is able to be diagonalized ($P^{-1}AP = D$, when D is diagonal). Know how to construct D and P. Orthogonal diagonalizability as well.

Fourier Series

(a) Inner products of functions, and extension of the Linear Algebra terminology to function spaces. For instance, orthogonality, linear combination, etc.

(b) Orthogonal sets of functions. Definition and method to show a given set is orthogonal (and possibly orthonormal). Know the procedure for obtaining a series expansion for a given function $f(x)$ with respect to an infinite orthogonal set of functions. That is, for an orthogonal set (basis) $\{\phi_i(x)\}_{i=1}^{\infty}$, you should be able to find the coefficients $\{c_i\}_{i=1}^{\infty}$ such that

$$f(x) = \sum_{i=1}^{\infty} c_i \phi_i(x).$$

(c) Fourier series. Know the definition, the orthogonal basis being used, and the interval where the expansion holds. **Have the formula memorized.**

(d) Convergence of the Fourier series to the actual function $f(x)$. For what x do the two coincide (Answer: points of continuity)? When they don't coincide, what is the value of the Fourier series at the point x (Answer: $\frac{f(x-)+f(x+)}{2}$)?. Know how to represent both graphically.

(e) Even and odd functions, and their corresponding Fourier series as cosine and sine series, respectively. Extensions to Fourier series on $[0, L]$ (as opposed to $(-p, p)$). That is, half-range expansions for even, odd, and identity reflections.

(f) Complex Fourier series. Equivalent formulation in terms of complex exponentials ($e^{in\pi x/p}$), as opposed to sine and cosine series. Know all the formulas, and the relation to the frequency spectrum of a function.

<u>PDEs and Boundary Value Problems</u>

(a) Boundary value problems for ODEs. Definition of an eigenvalue and corresponding eigenfunction of an ODE with parameter λ. How boundary conditions restrict the possible values of λ where non-trivial solutions exist. You should be familiar with second order, constant coefficient, homogeneous ODEs for this material. Also, the Sturm-Liouville formulation, and what is says about eigenfunctions.

(b) Basics of PDEs. How problems are formulated (usually initial and boundary conditions). Linear vs. nonlinear, priniciple of superposition, and classification of second order, constant coefficient, linear PDEs (hyperbolic, parabolic, and elliptic).

(c) Method of separation of variables. What it says, and how it (often) reduces a PDE to a system of ODEs. Furthermore, the method of solving these ODEs (in terms of the separation constant λ), and combining them to form a solution of the original PDE.

(d) Heat equation. Understand where the equation comes from, and what it represents. Formulation as a boundary value problem with initial conditions. Especially the different types of boundary conditions (Dirichlet, Neumann, mixed), and what they represent physically. You will **NOT** need to **solve** the BVP for the heat equation for this exam, but you may need to set one up.