MATH 336: Homework #8

Due: Tuesday, April 5, 2016

Solve the below problems concerning ordinary differential equations. A (possibly improper) subset of them will be graded. All calculations should be done analytically, unless marked with an (M). (M) problems require the use of MATLAB. ES denotes the online lecture notes.

1. (20 points) (ES, p.148-149, #7) Problem 7 in the ODE6 section of the notes (end of chapter 2). A few notes:
   (a) Parts (b)-(d) are completely qualitative, and just require a sentence of explanation (i.e. no calculations are required). **Enhancer** here means that that the mechanism of R (binding to promotor site on DNA D) increases the translation of protein P; a **repressor** does the opposite. Since P is completely translated by the mRNA M, it is equivalent to examining whether R increases (enhancer) or decreases (repressor) the transcription of M. Thus, there are only two reactions you really need to analyze: the first and the last. Of course, please give a brief reason for your answers here.
   (b) Resist the urge to use MATLAB for part (e). It is good practice in solving linear systems, which you may need to do for the exam.
2. (20 points) (ES, p.150, #10) Problem 10 in the ODE6 section of the notes (end of chapter 2).
3. (20 points) Consider the cooperative reaction mechanism:

$$nS + E \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C \xrightarrow{k_2} P + E$$

   In class, we saw that this reaction scheme can be used to biologically explain sigmoidal kinetics, **if** $n > 1$, n an integer (remember the overall kinetics are defined in terms of the rate of production, that is production of P in the above network). Show this analytically using calculus. More precisely, show that the production rate has an inflection point if and only if $n > 1$.

Furthermore, find this inflection point (as a function of the substrate S concentration). *Note:* You do **NOT** need to re-derive the production rate, but instead just use the formula presented in class (or in the notes).

4. (20 points) (M) This is a direct continuation of problem 5 in the last homework (HW # 7). There, we solved the enzyme reaction network equations exactly. We would now like to investigate the error incurred using the quasi-steady state (aka Michaelis-Menten) approximation.

   Using the same rate constants, solve the reduced "system" (it is really just one equation) using MATLAB, and plot the results of of all four chemical species (S, E, C and P) on the same set of axes. Include in your output the plot of last week's MATLAB problem (the exact solution of the same system), and compare the results.

   For example, answer the following questions: How good is the approximation? For what times is it good/bad? Do both systems approach the same steady state? Is it good for most times, or only for a small amount of time?

   Hint: Some of your initial conditions will have to be changed (or disregarded), since we making an approximation at a certain timescale. Be careful to understand which one(s). *Note:* Please check the solution for HW # 7, so as to make sure your code/output is correct; otherwise the comparison will be meaningless. All of the code is provided in the solution to HW # 7 on SAKAI.

5. (20 points) (ES, p.152, #4) Problem 4 in the ODE7 section of the notes (end of chapter 2). Note that "hyperbolic" vs. "sigmoidal" again refers to the shape of $c(\infty)$, in the same way we discussed for cooperative reactions.