MATH 336: Homework #6

Due: Thursday, March 10, 2016

Solve the below problems concerning ordinary differential equations. A (possibly improper) subset of them will be graded. All calculations should be done analytically, unless marked with an (M). (M) problems require the use of MATLAB. ES denotes the online lecture notes.

1. (20 points) (ES, p.142, #1) Problem 1 in the ODE5 section of the notes (end of chapter 2).

2. (20 points) (ES, p.142, #3) Problem 3 in the ODE5 section of the notes (end of chapter 2).

3. (20 points) (ES, p.142, #4) Problem 4 in the ODE5 section of the notes (end of chapter 2). Note:

   (a) See the previous problem for an introduction to vital dynamics.

   (b) *Your answers to the existence of steady states and stability will depend on parameter values. Thus, you should state cases for the different parameter regimes.*

4. (20 points) (ES, p.143, #6) Problem 6 in the ODE5 section of the notes (end of chapter 2). Note:

   (a) You will need to compute a Jacobian matrix for this 3×3 system. You **cannot** use the trace-determinant diagram used for 2×2 systems (it no longer applies). Thus, you must find the eigenvalues, and examine them directly to determine stability.

   (b) *As in the previous problem, your answers to the existence of steady states and stability will depend on parameter values.*

5. (20 points) (M) Consider the SIRS model

$$\frac{dS}{dt} = -0.003SI + 0.5(1000 - S - I)$$
$$\frac{dI}{dt} = 0.003SI - I.$$

Note that this set of equations corresponds to parameter values $N = 1000, \beta = 0.003, \nu = 1, \gamma = 0.5$. Use MATLAB to numerically solve the above system of ODEs on the time interval $[0, 20]$, with initial conditions $S(0) = 999$, $I(0) = 1$. Please **don't** hand-in a list of numeric values for the solutions; simply plot the curves $S(t), I(t), R(t) = N - S(t) - I(t)$ on the same set of axes. That is, you should plot S vs. t, I vs. t, and R vs. t on one plot. You may use the following MATLAB code as a template; note that like last time, it is not complete. It is fairly well-commented, with some comments containing hints.

```
%Constants defined for SIRS model
beta=;
nu=;
gamma=;
N=;

%Define the initial conditions
S0=;
I0=;
t0=0;
tF=20;
initials=[S0,I0];

%Now solve the ode and plot the trajectories from t=0 to t=20 (arbitrary)
[T,SI]=ode45(@rhsHw6No1,[t0 tF],initials,[],beta,gamma,nu,N);
S=SI(:,1);  %S population is first column of SI
I=;         %I population is second column of SI
R=;         %R is the "leftover" population from N (which is constant).
            %Note that you can be naive here, even if the dimensions...
            %...don't seem to make sense.

%Plot the solutions
figure(1)
plot(T,S,'-b','LineWidth',2);  %Plot of susceptibles
hold on;
plot(,'--r','LineWidth',2);    %Plot of infected
plot(,'-.k','LineWidth',2);    %Plot of recovered
legend('S(t)','I(t)','R(t)');
xlabel('t');
ylabel('number of individuals');
title('SIRS model dynamics');
```

You must also define the vector field (i.e. the RHS of the ODE) in a separate *m-file*, entitled *rhsHw6No1.m*. The skeleton for that file is below. Note that you must fill-in the second component for it to run.

```
function dSIdt=rhsHw6No1(t,SI,beta,gamma,nu,N)

    S=SI(1) % corresponds to susceptible
```

```
    I=SI(2)    % corresponds to infected

    %The first entry corresponds to the equation for dS/dt, while the ...
    % ... second is for dI/dt

    dSIdt=[-beta*S.*I+gamma*(N-S-I);];
end
```