MATH 336: Homework #10

Due: Tuesday, April 26, 2016

Solve the below problems concerning Bendixson's criterion and partial differential equations. A (possibly improper) subset of them will be graded.

1. (15 points) (ES, p.157, #10) Problem 10 in the ODE8 section of the notes (end of chapter 2).
2. (15 points) (ES, p.157, #12) Problem 12 in the ODE8 section of the notes (end of chapter 2).

Note that for all remaining problems, we consider the one-dimensional conservation of mass equation, with **advection only**, so that

$$J(x,t) = c(x,t)v(x,t), \tag{1}$$

where v is the velocity of the medium. I began discussing this on Tuesday, but just assume this form for now, and use the equation derived with this form for J to answer the following problems.

3. (15 points) Assume that $v(x,t)$ is constant in space and time, so that $v(x,t) \equiv v$, and that $\sigma(x,t) := \lambda c(x,t)$ (i.e. exponential growth/death), for $\lambda \in \mathbb{R}$. Show that

   $$h(x,t) := e^{\lambda t} f(x - vt)$$

   is a solution to the conservation of mass equation (advection equation in this case), for any differentiable function $f : \mathbb{R} \to \mathbb{R}$. (We will do the other direction on Thursday (4/21).)
4. (20 points) (ES, p.198, #1) Problem 1 in the PDE1 section of the notes (end of chapter 3). *Hint:* See a similar example on page 166.
5. (15 points) (ES, p.198, #2) Problem 2 in the PDE1 section of the notes (end of chapter 3). *Hint:* Is v constant here? Be careful.
6. (20 points) (ES, p.199, #6, parts (3) and (6) **only**) Problem 6 in the PDE1 section of the notes (end of chapter 3). You only need to do cases (3) and (6), for both parts of (a) and (b). That is, don't write up all 7 cases listed in the notes.