MATH 336: Homework #1

Due: Tuesday, February 2, 2016

Solve the below problems concerning difference equations. A (possibly improper) subset of them will be graded. All calculations should be done analytically, unless marked with an (M). (M) problems require the use of MATLAB. ES denotes the online lecture notes.

1. (25 points) (partly taken from ES, p. 22, #2) Suppose you observe a frog embryo in which the cells divide roughly every half hour. We start with one cell at time 0.
   (a) Write down an equation for P_{n+1} in terms of P_n that models this situation. Explain what time units you are using, and what is the initial value P_0.
   (b) How many cells are there after 3 hours?
   (c) (M) Graph the time-series for the first 8 iterates of your equation from part (a). That is, plot P_n vs. n for $n = 1, 2, \ldots, 8$. *Hint:* The (unfinished) main code is below:

```
N=0:1:8;
P=;  %Write down the solution for the above model
     %It should be exponential, and you should use the .^ command
     %Note that you can also use a "for" loop if you'd prefer
plot(N,P);
```

   However, please label your axes (google MATLAB documentation and see Review Homework), by adding to the above fragment. Again, it is *highly* recommended to create an "m-file" to run all MATLAB code in this course.

2. (15 points) (ES, p.23, #6) Explain why the model $\Delta P = rP$ cannot be biologically meaningful for describing a population, if $r < -1$.

3. (10 points) (ES, p. 25, #5) Problem 5 in the SDE2 section of the notes (end of chapter 1). Note that you should just print the page and draw the cobwebbing on the provided plots.

4. (20 points) (partly taken from ES, p. 26, # 1 and 2) Consider the difference equation

$$P_{t+1} = P_t + \alpha P_t - \beta P_t^2$$

   (a) Find all steady states of the above difference equation.

   (b) Determine the stability of each steady state found in part (a). Note that, since α and β are arbitrary, your answers will be algebraic conditions on the parameters (e.g. $P_* = 6$ is stable when $\alpha > \beta$, unstable when $\alpha < \beta$, but of course not this exactly).

5. (30 points) Consider the nonlinear equation for population growth

$$x_{n+1} = \frac{rx_n}{1 + x_n},$$

where $r > 0$.

   (a) Find all steady states, and state for what parameter values they exist.

   (b) Find the stability of the steady state(s), and discuss any bifurcations that occur.

   (c) Draw cobweb diagrams for each case that you identified in part (b).

   (d) (Hard) Find the explicit solution (dependent on initial value x_0). *Hint:* Make the change of variables $y_n := 1/x_n$, and write a difference equation for y_n, instead of x_n. That is, there should be no mention of any x_n terms in the new equation. Next, guess a solution of **the new equation** of the form $y_n = A\lambda^n + B$. Identify A, λ, and B (one or more could be arbitrary). Remember, you can choose anything (nontrivial) you'd like for A, λ, and B that will be make the resulting equation identically zero. Does the solution agree with what you found in part (b)? Explain.