MATH 350: Linear Algebra

Quiz 5

NAME: ________________________________ Date: December 10, 2018

Solve the following problems on this sheet of paper. No calculators or other electronic devices are permitted. **There is a problem on the back!**

1. (6 points) Recall that we showed in class that, for a finite-dimensional subspace U of an inner product space V, that there exists the direct-sum decomposition

$$V = U \bigoplus U^{\perp}.$$

That is, for each $v \in V$, there exists unique $u \in U, w \in U^{\perp}$ such that

$$v = u + w.$$

For such a decomposition of v, define the linear operator $P_U : V \to V$ by

$$P_U(v) := u.$$

Note that this is just the orthogonal projection of V onto U, so that $P_U(v) \in U$. Show that, for each $v \in V$,

$$||v - P_U(v)|| \leq ||v - u||,$$

for all $u \in U$. That is, the projection satisfies a natural minimization problem of finding $u \in U$ such that $||v - u||$ is smallest.

2. (4 points) Prove or disprove: there is an inner product on $\mathbb{R}^2$ such that the associated norm is given by

$$||(x_1, x_2)|| = |x_1| + |x_2|.$$