MATH 350: Linear Algebra

Quiz 3

NAME: _____ Date: November 1, 2018

Solve the following problems on this sheet of paper. No calculators or other electronic devices are permitted.

1. (3 points) Are $M_{2\times 2}(\mathbb{R})$ and $P_3(\mathbb{R})$ isomorphic vector spaces over $\mathbb{R}$? Provide justification either way.

2. (7 points) Consider the linear transformation $T : \mathbb{R} \to \mathbb{C}^2$ defined by

$$T(x) = \begin{pmatrix} x \\ 2x + ix \end{pmatrix}$$

Here $i = \sqrt{-1}$ (as usual). Recall that, for this to be well-defined, $\mathbb{R}$ and $\mathbb{C}^2$ are viewed as vector spaces over $\mathbb{R}$. Consider the following ordered bases β, γ for $\mathbb{R}$ and $\mathbb{C}^2$, respectively:

$$\beta = \{1\},$$
$$\gamma = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ i \end{pmatrix} \right\}.$$

   (a) What is the size of the matrix $[T]_\beta^\gamma$?

   (b) Find $[T]_\beta^\gamma$.

   (c) Using your result from (b), find the rank of T.