MATH 350: Homework #7

Due: Thursday, November 1, 2018

Solve the below problems concerning coordinates, elementary matrices, and determinants. A (possibly improper) subset of them will be graded. All calculations should be done analytically.

1. Find the change of coordinate matrix that changes β'-coordinates into β-coordinates, where

$$\beta = \left\{ \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \end{pmatrix} \right\},$$
$$\beta' = \left\{ \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \end{pmatrix} \right\}.$$

2. Let T be the linear operator on $P_1(\mathbb{R})$ ($T : P_1(\mathbb{R}) \to P_1(\mathbb{R})$) defined by

$$T(f(x)) = f'(x).$$

Let β be the standard ordered basis for $P_1(\mathbb{R})$, and $\beta' = \{1 - x, 1 + x\}$ another ordered basis. Use the change of coordinate matrix Q to find $[T]_{\beta'}$. *Note:* You will have to find the inverse of a 2×2 matrix.

3. Write the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 3 & 0 & 0 \end{pmatrix}$$

as the product of elementary matrices.

4. **True or False:** Assume the A and B are matrices related by an elementary row operation. A and B may have different rank. Provide justification.

5. Consider the linear transformation $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ defined by

$$T(f(x)) = f''(x) + 2f'(x) - f(x).$$

Show that T is invertible. Using the standard basis for $P_2(\mathbb{R})$, find a representation for T^{-1}. *Note:* I am not asking you to find the a matrix representation for T^{-1}, but precisely $T^{-1}(f(x))$, for $f(x) \in P_2(\mathbb{R})$. Of course, to get this representation, you can use a matrix and theorems.

6. Prove that $\det(AB) = \det(A)\det(B)$ for any $A, B \in M_{2\times 2}(F)$.

7. Find the value of k that satisfies the following equation:

$$\det\begin{pmatrix} b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \\ a_1 + c_1 & a_2 + c_2 & a_3 + c_3 \\ a_1 + b_1 & a_2 + b_2 & a_3 + b_3 \end{pmatrix} = k \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$