MATH 350: Homework #2

Due: Monday, September 24, 2018

Solve the below problems concerning linear indepedence, bases, and dimension (mainly Sections 1.5-1.6). A (possibly improper) subset of them will be graded. All calculations should be done analytically

1. Find a set of linearly independent diagonal matrices that generate the vector space of 3×3 diagonal matrices over $\mathbb{R}$.

2. Let V be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. Show that is S_1 is linearly dependent, then so is S_2.

3. Find a basis for the following subspace of real polynomials:
$$W = \{f(x) \mid f(x) = ax^3 + bx^2 + cx + d,\ f(1) = 0\}$$

4. Is $\{1 + 2x + x^2, 3 + x^2, x + x^2\}$ a basis for $P_2(\mathbb{R})$? Justify your answer.

5. The set of all skew-symmetric matrices is a subspace of $M_{n\times n}(\mathbb{R})$ (recall that a matrix $A \in M_{n\times n}(\mathbb{R})$ is skew-symmetric if and only if $A^T = -A$). Find a basis for this subspace. What is its dimension?

6. The set of solutions of the system of equations
$$\begin{aligned} x - 2y + z &= 0 \\ 2x - 3y + z &= 0 \end{aligned}$$
is a subspace of $\mathbb{R}^3$. Find a basis for it, and its dimension.

7. Show that if W is a subspace of a finite-dimensional vector space V such that $\dim(W) = \dim(V)$, then $W = V$.