MATH 350: Homework #1

Due: Monday, September 17, 2018

Solve the below problems concerning abstract vector spaces (mainly Sections 1.1-1.5). A (possibly improper) subset of them will be graded. All calculations should be done analytically

1. Let $S = \{0, 1\}$, and $F = \mathbb{R}$. In $\mathcal{F}(S, R)$, show that $f = g$ and $f + g = h$, where

$$\begin{aligned} f(t) &= 2t + 1 \\ g(t) &= 1 + 4t - 2t^2 \\ h(t) &= 5^t + 1. \end{aligned}$$

2. A real-valued function f defined on the real-line is called an **even function** if $f(-t) = f(t)$ for all $t \in \mathbb{R}$. Prove that the set of even functions defined on the real line with the operations of addition and scalar multiplication normally defined (pointwise) is a vector space.

3. Show that if V is a vector space, and $v \in V$ and $c \in F$ are such that $cv = 0$, then either $c = 0 \in F$ or $v = 0 \in V$.

4. Is $W = \{(a_1, a_2, \ldots, a_n) \in F^n \,|\, a_1 + a_2 + \cdots + a_n = 1\}$ a subspace of F^n? Why or why not?

5. Prove that $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is linear independent over $\mathbb{R}$ but linearly dependent over $\mathbb{Z}_2$.

6. For following polynomials in $P_3(\mathbb{R})$, can p_1 be expressed as a linear combinations of p_2 and p_3?

$$\begin{aligned} p_1(x) &= x^3 + x^2 + 2x + 13 \\ p_2(x) &= 2x^3 - 3x^2 + 4x + 1 \\ p_3(x) &= x^3 - x^2 + 2x + 3 \end{aligned}$$

   Justify your answer.

7. Show that

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$M_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$M_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

span the space of all symmetric 2×2 matrices.