# Final Exam Info

**Date:** December 18th, 2018

**Location:** Normal classroom (SEC-203, Busch Campus)

**Time:** 12:00 - 3:00 pm

**Notes:**

1. The exam will be cumulative, but will be slightly more focused on the material since Exam 2 (invariant subspaces, Cayley-Hamilton, Jordan canonical form, and inner product spaces).

2. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.

3. I will hold extra office hours on Friday (12/14) afternoon; I plan to be in my office from 2-4 pm (Hill 216) on December 14th. I will also have office hours on Monday, December 17th from 11-2. If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.

4. I have designed the exam to take approximately two hours, but you may stay the full three if you'd prefer.

**Suggestions:**

1. Understand all previous exam questions.

1. Read over covered sections in the textbook, as well as notes from class.

2. Understand all assigned homework questions and quizzes. Solutions to both are available on Sakai.

3. Do the review problems posted on the course site.

3. Solve other (unassigned) homework questions from the same section of the textbook.

**Material:**
All material covered in the course is fair-game for the exam. Regarding the material from the first two exams, I suggest looking at their respective information sheets to get an extensive list of topics to review. For the new material, see the below list and the Course Calendar. As usual, this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for the complete set of topics for the course.

Invariant Subspaces and the Cayley-Hamilton Theorem (Section 5.4)

(a) Examples of invariant subspaces with respect to a linear operator (for example, $E_\lambda, N(T), \{0\}$). How to show a subspace is invariant.

(b) For finite-dimensional linear operators, what this means with respect to a matrix representation (block-diagonal form, if we can form a direct-sum decomposition of invariant subspaces).

(c) Properties of characteristic polynomial for linear operator restricted to an invariant subspace.

(d) Cyclic subspaces generated by an element $x \in V$. How to find a basis and the characteristic polynomial restricted to this subspace.

(e) For a polynomial $f(t)$, the meaning of the linear operator $f(T)$, when $T \in \mathcal{L}(V)$. Basic properties.

(f) Cayley-Hamilton Theorem. How to prove, and why is it useful.

Jordan Canonical Form (Sections 7.1-7.2)

(a) Be able to define the form precisely. What is a general Jordan block? What happens in the transformation is diagonalizable?

(b) What are the conditions for it to be guaranteed to exist?

(c) Generalized eigenvectors and generalized eigenspaces. How they are defined, and their basic properties.

(d) Basic method of computation. How to compute the Jordan form, and the Jordan basis, for a given linear operator $T$. Finding the similarity matrix $Q$ that transforms $[T]_\beta$ into Jordan form.

(e) Cycles and their length. Relation between cycles and a basis of the generalized eigenspaces. What they tell you about each Jordan block?

(f) Direct sum decomposition for generalized eigenspaces (remember, these are invariant subspaces).

(g) Relation between geometric and algebraic multiplicity of an eigenvalue. What each tells you about the generalized eigenspaces, cycle decomposition, and Jordan canonical form.

Inner Product Spaces (Sections 6.1-6.2)

(a) Definition of an inner product on a vector space (i.e. an *inner product space*).

(b) How an inner product spaces allows one to define geometric notions.

(c) Basic properties shared by all inner product spaces (Pythagorean Theorem, Parallelogram Law, Cauchy-Schwarz, etc.)

(d) Standard examples of inner product spaces ($\mathbb{R}^n, \mathbb{C}^n$ with the dot product, $C[0,1]$ with $\int_0^1 f(t)\bar{g}(t)\,\mathrm{d}t$, Frobenius inner product, etc.)

(e) Basics of complex numbers (complex conjugates, etc.), since this is needed in the definition of an inner product.

(f) How to define length and orthogonality (perpendicularity) in an inner product space.

(g) Orthonormality, and why it is useful.

(h) Gram-Schmidt Process: turning a linearly independent set of vectors to an orthogonal (and hence orthonormal by normalizing) set that has the same span. Computational aspects (i.e. be able to do this for specific inner product spaces).

(i) Orthogonal complement of a subspace $U$. Basically, if $U$ is a finite-dimensional subspace of an inner product space $V$, then $V = U \bigoplus U^\perp$. How to use this to define the orthogonal projection onto $U$ (note that we defined projections in the first part of the course, this is just a specific example).