Exam 2 Info

Date: November 15th, 2018

Location: Normal classroom (SEC-203, Busch Campus)

Time: In-class (10:20-11:40 am)

Notes:

1. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.
2. I will hold extra office hours on Wednesday (November 14th). I plan to be in my office from 2 pm - 4 pm (Hill 216). Feel free to stop by at any time in that interval (no need to email beforehand) with any questions you have. If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.
3. **You will be expected to prove a number of statements rigourously.** Review basic techniques seen in the class, homework, and in the textbook.

Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions and quizzes. Solutions to both are available on the Sakai.
3. Do the review problems posted on the course site.
3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material up to and including what will be covered on Monday's (November 12th) class (and not covered on Exam 1) is fair-game for the exam. Roughly speaking, this is all of the material on coordinates, matrix representations, isomorphisms, determinants, and eigenvalues/eigenvectors. Note that I quickly reviewed Chapters 3 and 4, since they contain material that should be familiar from your first course on Linear Algebra. **You are responsible for this material.** Also, I did not cover Section 2.6, and material from this section will **not** be included in Exam 2. Some key topics to review are given below. But be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered. Note also that although I will not be explicitly testing on material appearing before Exam 1, you will need to be comfortable with many of the concepts (linear transformations, vector spaces, direct sums, etc.)

Linear Transformations (Chapter 2, Sections 2.2-2.5)

(a) Coordinate representations with respect to an ordered basis for a finite-dimensional vector space. Both for vectors and linear transformations. How to compute, and basic properties. Understand how operations on coordinates relate to operations on the vector spaces themselves. For instance, matrix multiplication corresponds to (linear) function composition. Translating between coordinates and vectors should is very fundamental, and you should be comfortable moving back and forth between the two notions.

(b) Change of coordinates. How coordinate and matrices with respect to ordered bases change, as the bases vary. Change of coordinate matrix: how to find it, and what it does.

(c) Invertibility and isomorphisms. Basic definitions, and how to check if linear transformation is invertible. How this relates to invertibility of matrices, and using matrix inverses to find inverse transformations. When are two vector spaces (over the same field) isomorphic? What does this mean? Constructing isomorphisms and checking if a map is an isomorphism.

Matrix Operations/Systems of Equations (Chapter 3)

(a) Elementary row operations and how they relate to elementary matrices.

(b) Rank and nullity of a matrix.

(c) Basic properties of rank and nullity (for example, how is $\mathrm{rank}(T)$ related to $\mathrm{rank}(TS)$, for linear transformations T and S?)

(d) Relation of rank and nullity to solving systems of equations.

(e) Invertibility of matrices. Be able to compute A^{-1} via elementary row operations.

Determinants (Chapter 4)

(a) Definition of determinants. Know how to compute from definition (cofactor expansion).

(b) How elementary row operations affect the determinant.

(c) Basic properties of determinants. For example, when it makes sense, $\det(AB) = \det(A)\det(B)$. What about the determinant of a triangular matrix? You should have memorized (and even be able to prove) this basic (and useful!) properties.

(d) How determinants relate to invertibility of a linear transformation.

(e) Be comfortable with basic calculations. I won't ask you to compute the determinant of a 10×10 matrix, but you should be able to quickly compute for a 2×2 or 3×3.

Eigenvalues/Eigenvectors and Diagonalizability (Chapter 5, Sections 5.1-5.2)

(a) Definition of an eigenvalue/eigenvector of a linear map. Note: not just for a matrix, but a general linear operator, on a (possibly infinite dimensional) vector space.

(b) Invariant subspaces of a linear operator. Know how they relate to eigenvalues. What are the invariant subspaces related to the eigenvalues?

(c) Compute eigenvalues/eigenvectors using matrix representation on a finite-dimensional space.

(d) Diagonalizability. When is a linear operator diagonalizable? How do we find a basis where its representation is diagonal? What are the necessary and sufficient conditions for this? Note that you should be able to prove all statement!

(e) Testing for diagonalizability. How do geometric and algebraic multiplicities relate? What is a natural direct sum decomposition of V if T is diagonalizable?