Exam 1 Info

Date: October 11th, 2018

Location: Normal classroom (SEC-203, Busch Campus)

Time: In-class (10:20-11:40 am)

Notes:

1. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.
2. I will hold extra office hours on Wednesday (October 10th). I plan to be in my office from 11 am - 3 pm (Hill 216). Feel free to stop by at any time in that interval (no need to email beforehand) with any questions you have. If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.
3. **You will be expected to prove a number of statements rigourously.** Review basic techniques seen in the class, homework, and in the textbook.

Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions and quizzes. Solutions to both are available on the Sakai.
3. Do the review problems posted on the course site.
3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material up to and including what will be covered on Monday's (October 8th) class is fair-game for the exam, up to and including coordinates and matrix representations of linear transformations (again, you will see this on Monday). Roughly speaking, this is all of the material on basic properties of vector spaces seen in Chapter 1, and introductory material on linear transformations from Chapter 2. Note that we did not cover Section 1.7, and this material will **not** be included on the exam. Some key topics to review are given below. But be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered.

<u>Vector Spaces (Chapter 1)</u>

(a) Basic definition of a vector space. You should be able to prove basic properties just using the abstract vector space definition (and not, for instance, that fact that you are in $\mathbb{R}^2$ or $\mathbb{R}^3$).

(b) Examples of vector spaces. Be comfortable with the standard examples $F^n, \mathbb{R}^n, \mathbb{C}^n, P_n(F), P(F), M_{m\times n}(F)$ and others introduced in class and the textbook.

(c) Subspaces. Definition and how to check a given set is a subspace.

(d) Linear combinations, linear independence, spanning, basis, and dimension. These are all interrelated concepts. Understand how they fit together.

(e) Show that a set of vectors is (a)linearly independent/dependent, (b)spanning, (c)basis, etc. Basic computational techniques in diffeent spaces related to these concepts.

(f) Utility of a basis for a finite-dimensional vector space. Why is it useful? What does it allow us to do? This is the main tool for finite-dimensional spaces, so understand this concept well.

(g) Sums and direct sums of vector spaces. How sums generalize a union. Expressions for sums and direct sums of subspaces. Know when a sum is actually direct.

(h) Proofs of basic theorems. I may ask you to repeat a proof presented in class or in the textbook. Make sure to be familiar with the basic idea of all arguments presented.

Linear Transformations (Sections 2.1 and 2.2)

(a) Defintion of a linear transformation. How to show a function is linear. Basic properties of such transformations (for example, $T(0) = 0$ if T is linear).

(b) Standard examples of linear transformations. How to represent as a matrix for transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$.

(c) Definitions and basic properties of the null space $N(T)$ and range $R(T)$ of a linear transformation. Again, basic proofs that they are subspaces.

(d) Computing bases for $N(T)$ and $R(T)$ given T. Again, think standard examples, like Euclidean, matrix, polynomial spaces, etc.

(e) Rank and nullity of a linear transformation. $\text{rank}(T) = \dim(R(T)), \text{nullity}(T) = \dim(N(T))$.

(f) Dimension theorem relating rank and nullity of a linear transformation. How to use to find basic information about linear map.

(g) Definition of one-to-one $(1-1)$ and onto for a general function. How being $1-1$ relates to the null space for a linear transformation.

(h) Coordinates of a vector with respect to an ordered basis.

(i) Matrix representation of a linear transformation between finite-dimensional vector spaces.