

MATH 252: Elementary Differential Equations

Quiz 6

NAME: _____

Date: November 2, 2017

Solve the following problems on this sheet of paper. **Note that there is a problem on the back.**
No calculators or other electronic devices are permitted.

1. (6 points) Solve the initial-value problem

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -4 & 1 \\ 2 & -3 \end{pmatrix}, \quad \mathbf{Y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

2. (4 points) Note that this problem is modified slightly from the notes, to reduce the number of computations. Consider the matrix

$$A = \begin{pmatrix} 5 & 6 \\ -1 & -2 \end{pmatrix}$$

In the following, you may use, without justification, the fact that A has the following eigenvalue/eigenvector pairs:

$$\lambda_1 = 4, \quad \mathbf{v}_1 = \begin{pmatrix} -6 \\ 1 \end{pmatrix},$$
$$\lambda_2 = -1, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

- (a) Find an invertible matrix S and diagonal matrix Λ such that

$$A = S\Lambda S^{-1}.$$

Your answer should consist of the **three** matrices S, S^{-1} , and Λ .

- (b) Use your result from part (a) to find the matrix exponential e^{At} . **You may leave your answer as a product of matrices** (i.e. do not calculate the product to obtain a single matrix).