MATH 252: Elementary Differential Equations

Quiz 5

NAME: ——————————————————————— Date: October 26, 2017

Solve the following problems on this sheet of paper. **Note that there is a problem on the back.** No calculators or other electronic devices are permitted.

1. (4 points) Rewrite the following linear system in **component** form:

$$\left(\begin{array}{c} \frac{dx}{dt} \\ \frac{dy}{dt} \end{array} \right) = \left(\begin{array}{cc} 0 & \beta \\ \gamma & -1 \end{array} \right) \left(\begin{array}{c} x \\ y \end{array} \right)$$

That is, write the ODEs for the components x and y.

2. (6 points) Consider the system

$$\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}, \tag{1}$$

where

$$A = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix}$$

(a) Show that

$$\mathbf{Y}_1(t) = (e^{-3t} - 2e^{-4t}, e^{-3t} - 4e^{-4t}),$$
$$\mathbf{Y}_2(t) = (2e^{-3t} + e^{-4t}, 2e^{-3t} + 2e^{-4t})$$

are both solutions of the above system (1).

(b) Calculate $\mathbf{Y}_1(0)$ and $\mathbf{Y}_2(0)$ and show that they are linearly independent.

(c) Use the above to solve the IVP consisting of system (1) with initial condition

$$\mathbf{Y}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$