Final Exam Info

Date: December 20th, 2017

Location: Normal classroom (TIL-226, Livingston Campus)

Time: 12:00 - 3:00 pm

Notes:

1. The exam will be cumulative, but will be slightly more focused on the material since Exam 2 (trace-determinant plane, non-homogeneous equations, and nonlinear systems).
2. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.
3. I will hold extra office hours on Tuesday afternoon. I plan to be in my office from 2-5 pm (Hill 216) on December 19th. If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.
4. I have designed the exam to take approximately two hours, but you may stay the full three if you'd prefer.

Suggestions:

1. Understand all previous exam questions.
1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions and quizzes. Solutions to quizzes are available on Sakai.
3. Do the review problems posted on the course site.
3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material covered in the course is fair-game for the exam. Regarding the material from the first two exams, I suggest looking at their respective information sheets to get an extensive list of topics to review. For the new material, see the below list and the Course Calendar. As usual, this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for the complete set of topics for the course.

<u>Second-Order Homogeneous Equations (Section 3.7)</u>

(a) Solving second-order homogeneous equations. Again, know how to convert between systems and equations, and how the form of solutions for systems produces general solutions for equations.

(b) Basic properties of the solution of the damped harmonic oscillator, for different physical scenarios. What does overdamped, underdramped, and critically damped mean, and how does it relate to the eigenvalues of the equation?

(c) Trace-determinant plane. Using this to determine the type of phase portrait your system has. Also, curves traced out in systems with a parameter (i.e. $A = A(\alpha)$), and bifurcation values based on qualitative changes in the phase portrait.

Forcing, or Non-Homogeneous Equations (Sections 4.1-4.3)

(a) Finding the general solution of a first-order, non-homogeneous equation $\mathbf{Y}' = A\mathbf{Y} + \mathbf{b}(t)$, using matrix exponentials.

(b) Solving second-order non-homogeneous equations ($ay'' + by' + cy = g(t)$) using the method of undetermined coefficients, with forcing functions of the form $g(t) = e^{at}$ and/or $g(t) = A_0 + A_1 t + ... + A_n t^n$. Know how to compute cases when $g(t)$ **is** and **is not** a solution of the corresponding homogeneous equation.

(c) Solving second-order non-homogeneous equations with periodic forcing terms (e.g. $y'' + 2y' + 6y = 5\cos(3t)$. Using complexification to write the solution as a single sine or cosine ($y(t) = A\sin(\omega t + \phi)$, i.e. finding A, ω, and phase shift ϕ). Be able to analyze solutions and long-term behavior (e.g plot).

(d) Undamped second-order non-homogeneous equations and resonance. Forcing for the undamped oscillator, where the forcing frequency approaches the natural frequency (beating). Be able to find general solutions, and interpret qualitatively via a graph. Also be able to write the solution as a **product** of sines (or sines and cosines), and be able to find the slow beating frequency (and period) and more rapid frequency from this expression. Also be able to solve the case of resonance (forced frequency equaling the natural frequency) and analyze the corresponding behavior graphically.

Nonlinear systems (Sections 5.1, maybe ideas from other sections)

(a) Basic properties of nonlinear systems, including finding equilibrium solutions.

(b) Linearization. Using Jacobian analysis to find the phase portrait of a nonlinear system **locally near an equilibrium point.** Know all of the linear theory (i.e. Chapter 3), including how to draw phase portraits for any 2×2 matrix, and how this relates to the phase portrait for the nonlinear system. In particular, be able to find eigenvalues and eigendirections, as well as directions of rotations for spiral sources/sinks. Note that this picture is only local, and nonlinear systems gennerally don't possess straight-line solutions, although the stable/unstable directions are tangent to the eigendirections at the equilibrium. Also know when the analysis does and does **NOT** apply (*Answer:* Applies when corresponding linear system eigenvalues have **non-zero** real part.)

(c) Conserved quantities. Be able to show that a given function $H(x, y)$ is conserved for a given system of ODEs. Use this to plot trajectories in the phase plane. And use the actual ODEs to determine the direction of motion along these trajectories.

(d) Understanding the difference between linear and nonlinear systems, and why nonlinear systems are so hard, and no general theory exists. Be able to follow "leading problems" through some analysis of specific nonlinear systems (e.g. Problems 9, 10, 14 in **Exam 3 Review Problems**).