Exam 2 Info

Date: November 16th, 2017

Location: Normal classroom (TIL-226, Livingston Campus)

Time: In-class (5:00-6:20 pm)

Notes:

1. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.

2. I will hold extra office hours on Wednesday evening. I plan to be in my office from 3-6 pm (Hill 216). If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.

3. There will also be a review held by the TA Sam Braunfeld, with an exact date/time and location TBD. Information will be provided soon. He will plan to go over the review problems posted on the website, as well as answer any other questions.

Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.

2. Understand all assigned homework questions and quizzes. Solutions to quizzes are available on Sakai.

3. Do the review problems posted on the course site.

3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material up to and including what will be covered on Tuesday's (November 14th) class (and not covered on Exam 1) is fair-game for the exam. The last section, which I have NOT covered yet (Section 3.6), I will briefly discuss on Tuesday the 14th, and uses the system properties from Chapter 3 to study certain second-order equations (remember from Chapter 2 that there is a way to translate second-order equations to two-dimensional first-order systems). I will NOT go into detail, so I expect you to read and understand this section on your own. **I highly suggest reading this section before class, so you are familiar with it as soon as possible.** Thus, Exam 2 will roughly cover Sections 2.3, 2.4, 2.6, and 2.7 of Chapter 2, and sections 3.1-3.6 of Chapter 3. Note that some extra material not presented in the textbook was covered (phase-plane analysis and matrix exponentials), and you are responsible for that material as well (see resources LPP and ME). Some key topics to review are given below. But be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered.

First-order systems (Chapter 2)

(a) Harmonic oscillator. Know the basic physics, and both its representations (equation vs. system). Both the pure (simple harmonic motion) and damped version.

(b) Guess and check method for higher order equations. Guessing solutions (in particular as exponentials) of second (or higher) order equations.

(c) Decoupled systems. Be able to solve systems where (at least) one of the variables is independent of the other. This reduces the system to two first-order equations, which you can study via tools from Chapter 1.

(d) Phase plane methods. Writing trajectories as functions ($y = y(x)$) or level curves ($h(x, y) = c$) via the relation $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, which is separable, **sometimes**. Know how to use this method to obtain (and draw) trajectories in the phase plane.

(e) Existence and uniqueness for systems. Basically, for autonomous systems, trajectories cannot cross in the phase plane. Note that this is NOT true if the system is non-autonomous ($F = F(t, \vec{Y})$).

(f) Epidemic modeling. Basic assumptions behind SIR model, and techniques to understand dynamic and asymptotic behavior. Also interpreting mathematical results biologically.

Linear Systems (Chapter 3)

(a) Properties of linear systems. Specifically focus on the Linearity Principle (linear combination of solutions is again a solution, for **linear systems only**) and how to generate solutions of initial-value problems from known solutions using linearity.

(b) Conversion between second-order equations and first-order linear systems. Know and understand the basic physics of the damped harmonic oscillator, as both a system and an equation.

(c) Be able to find straight-line solutions of linear systems, if they exist. In other words, you should be very good at finding eigenvalues and corresponding eigenvectors of a matrix A.

(d) Finding the general solution of a system from straight-line solutions, and how to solve initial-value problems using row-reduction.

(e) Know how to solve the cases when a general solution from straight-line solutions do NOT exist. This includes the case of complex and repeated eigenvalues.

(f) Phase portraits. You should know how to plot phase portraits for **ANY** linear system, based on the eigenvalues and number of linearly independent eigenvectors. This included repeated roots as well as the case of 1 or 2 zero eigenvalues.

(g) Know the terminology, in particular, saddles, sinks, sources, spiral sinks, spiral sources, and centers. Know how to determine the direction of rotation for the case of complex and repeated eigenvalues.

(h) Plotting components of solutions based on the phase portraits and the general solution. As in Exam 1, you should be comfortable translating between phase portraits (y vs. x) and components (x vs. t, y vs. t).

(i) Solving second-order homogeneous equations. Again, know how to convert between systems and equations, and how the form of solutions for systems produces general solutions for equations.

(j) Basic properties of the solution of the damped harmonic oscillator, for different physical scenarios. What does overdamped, underdramped, and critically damped mean, and how does it relate to the eigenvalues of the equation?

(k) Matrix exponentials. How to find e^{At} for any 2×2 matrix A, and how to use it to solve ODEs. In particular for distinct eigenvalues, but also understand the **definition** as an infinite series.