Exam 1 Info

Date: October 12th, 2017

Location: Normal classroom (TIL-226, Livingston Campus)

Time: In-class (5:00-6:40 pm)

Notes:

1. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.
2. I will hold extra office hours on Wednesday morning (October 11th). I plan to be in my office from 11 am - 2 pm (Hill 216). If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up. I will also probably have (slightly) elongated office hours on Thursday morning, but I will announce these later.
3. There will also be a review held by the TA Sam Braunfeld on Tuesday October 10th, with a location and time TBD. He will plan to go over the review problems posted on the website.

Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions and quizzes. Solutions to quizzes are available on the Sakai.
3. Do the review problems posted on the course site.
3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material up to and including what was covered on Thursday's (October 5th) class is fair-game for the exam, up to and including rewriting higher-order equations as first-order systems. Roughly speaking, this is all of the material from one-dimensional ODEs, and the introduction to system techniques from Chapter 2. Note that many sections were skipped in Chapter 2, and these will **not** be included on the exam. Some key topics to review are given below. But be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered.

<u>Single differential equations (Chapter 1)</u>

(a) Modeling via differential equations, especially exponential and logistic growth, and predator-prey systems (note that this is a system). You should be comfortable translating basic assumptions into mathematical expressions and differential equations. Also, interest rate and mixing (tank) problems are important (see the textbook).

(b) The concepts of a general solution and initial-value problem (IVP). In particular, how you get a solution to an IVP via the general solution.

(c) Separation of variables. This is the most important technique you have for actually solving differential equations at the moment. Know when and how to apply it to solve IVPs and find general solutions. Also, know when it doesn't work to find the general solution (think $y' = y^2$, with initial condition $y(0) = 0$).

(d) Slope fields. How to plot slope fields (by hand) for relatively simple differential equations $y' = f(t, y)$. Also know how to sketch solutions once you have a representative slope field, and what it tells you about trajectories geometrically.

(e) Euler's method. This is a numerical technique for approximating the solution of a differential equation. Be able to compute (by hand) the approximate solution to an IVP at any time T, given a step size $h = \Delta t$, and initial time t_0.

(f) Existence and uniqueness for differential equations. Know what conditions on $f(t, y)$ need to be satisfied for existence, and for uniqueness (note: they are different). Also be comfortable using the geometric consequences of uniqueness (namely, that solution curves cannot cross).

(g) Equilibria. What equilibria solutions are (constant solutions) and how to find them ($f(y) = 0$).

(h) Phase line. Be able to draw the phase line for a **autonomous** differential equation ($y' = f(y)$), given $f(y)$. Classification of equilibrium solutions via the phase line (sink vs. source vs. node). Furthermore, you should be able to draw solution curves, once you have the phase line.

(i) Bifurcations. What is a bifurcation, and how do we study them. In particular, varying parameters of an ODE and finding the parameter value where the dynamics change qualitatively. Also be able to produce a bifurcation diagram.

(j) Linear equations. What does it mean for a differential equation to be linear? Homogeneous vs. non-homogeneous, as well as constant-coefficient equations.

(k) Solution techniques for linear equations. You should be familiar with both methods: (i) guessing a particular solution form $y_p(t)$ given the forcing $b(t)$, and (ii) using the integrating factor.

First-order systems (Chapter 2)

(a) Geometric interpretation of a system $\vec{Y}' = F(\vec{Y})$ in vector form. Mainly that F is a vector field, which gives the tangent vector of solution curves. Be able to convert a system of equations into vector form, and vice versa.

(b) Be comfortable with the different representations of solutions. Know what the phase plane is, and how to translate between the phase plane and component form of solutions (this is y vs. x compared to x vs. t and y vs. t). Also a phase portrait as a representative diagram of trajectories in the phase plane (including equilibrium points).

(c) Equilibrium solutions for system. How to find them, and what types of trajectories they represent (just points!).

(d) Plotting the direction field of the right-hand side $F(\vec{Y})$ in the phase plane. You should be able to get a representative idea of the flow by plotting (by hand) the direction fields for relatively simply systems. Also, given a plot of the the direction field, you should be able to sketch (approximate) trajectories in the phase plane.

(e) Turning higher-dimensional equations into first-order systems. In particular, how to use the rule $v := y'$ to get a first-order system from a second-order equation. Be able to identify the components of $\vec{Y}$ and the vector field F of the transformed system.