Matlab Assignment #8

Pendulums and Nonlinear Systems

Use the files *DirectionField.m*, *ExampleDirectionField.m* and *H.m* for this assignment. Again, for all the Matlab plots you submit, use the function *title* to give the plot a title name which identifies which problem number.

A group of bored pendulum enthusiasts have called a meeting. They have read the book's discussion of pendulums on page 495-496 and know that they can model the angle and velocity (θ and v) of pendulums using differential equations of the form

$$\begin{aligned}\frac{d\theta}{dt} &= v \\ \frac{dv}{dt} &= -\frac{b}{m} - \frac{g}{l}\sin\theta\end{aligned}$$

where l is the length of the pendulum, m is the weight of the bob, g is the acceleration due to gravity, and b is the damping coefficient.

To bring some excitement into their humdrum lives, they've decided to submerge two differently sized pendulums in two liquids of differing viscosities. One pendulum is submerged into molasses, and no matter how hard they try, they are unable to get that pendulum to swing back and forth. The other pendulum is submerged in liquid nitrogen, and while it will start off oscillating, the pendulum will eventually come to a rest. These two pendulums can be modeled by the following nonlinear differential equations (in no particular order):

$$\mathbf{Y}_1{}'(\theta, v) = \begin{cases} v \\ -2v - 3\sin\theta \end{cases}$$

$$\mathbf{Y}_2{}'(\theta, v) = \begin{cases} v \\ -8v - 11\sin\theta \end{cases}$$

1. Find matrices A_1 and A_2 which are the linearization of the system Y_1 and Y_2 above at the point $(\theta, v) = (0, 0)$.

2. For each of the linear systems $\frac{d}{dt}\mathbf{Z} = A_1 \cdot \mathbf{Z}$ and $\frac{d}{dt}\mathbf{Z} = A_2 \cdot \mathbf{Z}$, determine what type of equilibrium the origin is.

3. Plot the direction fields for the nonlinear differential equation defined for $\mathbf{Y}_1{}'$ and $\mathbf{Y}_2{}'$ above. The range of your plot should be $[-4, 4] \times [-4, 4]$. (Hint: You will need to modify the file *H.m*)

4. Plot the direction fields for the linear differential equation $\frac{d}{dt}\mathbf{Z} = A_1 \cdot \mathbf{Z}$ and $\frac{d}{dt}\mathbf{Z} = A_2 \cdot \mathbf{Z}$.

5. Do your graphs from question 3 and question 4 look similar near the origin? (Hint: If the answer is no, then you've made a mistake.)

6. Which pendulum was submerged in liquid nitrogen, and which pendulum was submerged in molasses.