MATLAB Assignment #5

Linear Systems with Real and Complex Eigenvalues

You can refer to MATLAB files *ExampleLinearSystems.m* and *ExampleMatrix.m* from assignment #4 for help with the MATLAB component of this assignment. **Additionally**, for all the MATLAB plots you submit, use the function *title* to give the plot a title name which identifies which problem number it goes with.

For the differential equations given in questions #1-5, answer the following questions:

(i) What are the eigenvalues of the linear system? (You can use MATLAB and you do not need to show your work.)

(ii) What type of equilibrium is the origin? Is it a saddle, sink, source, center, spiral sink, or spiral source?

(iii) Plot the direction field of the differential equation.

1.

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \mathbf{Y}$$

2.

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -6 & 4 \\ -10 & 6 \end{pmatrix} \mathbf{Y}$$

3.

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 14 & 16 \\ -12 & -14 \end{pmatrix} \mathbf{Y}$$

4.

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 8 & 2 \\ -25 & -6 \end{pmatrix} \mathbf{Y}$$

5.

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -13 & -20 \\ 6 & 9 \end{pmatrix} \mathbf{Y}$$

6. Answer the questions below for the following system:

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & -5 \\ 5 & -3 \end{pmatrix} \mathbf{Y}$$

(i) Calculate the eigenvalues and corresponding eigenvectors of the linear system.

(ii) What type of equilibrium is the origin? Center, spiral sink, or spiral source?

(iii) What is the natural frequency of the oscillations in the system? What is the natural period length?

(iv) If $Y(0) = (1,1)$ then what is the derivative $Y'(0)$? Is the system spiral clockwise or counter-clockwise?

(v) Find two linearly independent solutions $Y_{re}(t)$ and $Y_{im}(t)$ such that the general solution of the system is of the form $Y(t) = k_1 Y_{re}(t) + k_2 Y_{im}(t)$.

(vi) Find the particular solution given by the initial condition $Y(0) = (1,1)$.

(vii) Using the formula you derived in part (vi), plot the $x(t)$ and $y(t)$ curves on the same graph for $0 \leq t \leq 3$. Make sure to indicate which curve is $x(t)$ and which curve is $y(t)$.

(Hint: You may need to use the "." symbol to perform element-wise multiplication of vectors. For example, to plot the function $x(t) = e^{-2t}\sin(4t)$, you could use the code below:)*

```
t_min = 0;
t_max = 3;
t_step = .01;
time = t_min:t_step:t_max;
x = exp(-2* time).*sin(4 * time);
plot(time,x);
```

(viii) On the $x \times y$-plane, plot the direction field of our linear system. On top of your direction field, plot the solution curve $Y(t) = (x(t), y(t))$ you found in part (vi). The dimensions of your graph should be $[-2,2] \times [-2,2]$ and you should plot the curve $Y(t)$ for $t \in [0,3]$.

(Hint: If your solution curve does not run parallel with the direction field, then you have made a mistake somewhere.)