MATLAB Assignment #2

Equilibria, Phase Lines and Bifurcations

1. In this assignment we study the one-parameter family of differential equations:

$$y'(t) = G_\alpha(y)$$

where we have defined $G_\alpha(y) := y^2 + 2y + \alpha$. For each of the sub-problems below, use the parameter values $\alpha = -3, 0, 1$, and 2.

a) Run the MATLAB code in *ExampleParameterizedSlopeField.m* to generate slope fields of the differential equation at the different parameter values. Print out the slope field for just one of the parameter values. (Choose your favorite one!)

b) For each parameter value, use the slope field to estimate all of the differential equation's equilibria within the range $-4 \le y \le 4$. Use the slope field to identify each equilibrium as either a (i) sink (ii) source or (iii) node.

c) Suppose that $y(t)$ is a solution to the differential equation and $y(0) = 0$. Predict the long term behavior of the function $y(t)$ for each parameter value. (i.e. Does it run off to positive/negative infinity, or does it approach an equilibrium? If it approaches an equilibrium, which one?)

d) For each parameter value, draw the phase line for the differential equation. Label each equilibrium with its value and type (i.e. sink, source, or node).

2. Identify the bifurcation value and draw the bifurcation diagram for the differential equation within the range $-4 \leq \alpha \leq 4$.