MATH 252: Elementary Differential Equations

Quiz 9

NAME: ______________________________ Date: December 8, 2016

Solve the following problems on this sheet of paper. **Note that there is a problem on the back.** No calculators or other electronic devices are permitted.

1. (6 points) Consider the nonlinear system

$$\frac{dx}{dt} = x(-x - y + 40)$$
$$\frac{dy}{dt} = y(-x^2 - y^2 + 2500)$$

This system has three equilibrium in the first quadrant: $(0,0), (0,50)$, and $(40,0)$.

(a) Calculate the Jacobian matrix $JF_{(x_0,y_0)}$ at an arbitrary equilibrium (x_0, y_0).

(b) Using (a), compute $JF_{(x_0,y_0)}$ at the three equilibrium in the first quadrant. *Hint:* $2500 = 50^2$.

(c) If possible, draw the **local** phase portrait for the nonlinear system near each of the three listed equilibrium. State the type of each point, and don't forget to include all pertinent information (directions of motion, correct tangencies, etc.) Put everything on one plot in the xy-plane.

2. (4 points) Consider the second-order non-homogeneous equation

$$\frac{d^2y}{dt^2} + 4y = \cos\left(\frac{9t}{4}\right).$$

(a) Find the general solution of the above equation.

(b) Using your result from (a), find the frequency of the beats.

(c) Similarly, determine the frequency of the rapid oscillations.

(d) Provide a qualitative sketch of the general solution $y(t)$. Note that no additional calculations are necessary here; all that is needed is the information obtained in (b) and (c), and the general theory about this type of forced equation.