

MATH 252: Elementary Differential Equations

Quiz 6

NAME: _____

Date: November 3, 2016

Solve the following problems on this sheet of paper. **Note that there is a problem on the back.**
No calculators or other electronic devices are permitted.

1. (5 points) Sketch the phase-portrait for the system

$$\begin{aligned}\frac{dx}{dt} &= 3x + 4y \\ \frac{dy}{dt} &= x.\end{aligned}$$

Note: For full credit, you must have the correct straight-line solutions, **if they exist.**

2. (5 points) Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & -5 \\ 3 & 1 \end{pmatrix} \mathbf{Y}.$$

- (a) Find the eigenvalues of the system. Note that they should be complex.
- (b) Determine the type of the phase portrait for this system (e.g. center, spiral sink, or spiral source). **Provide justification.**
- (c) Determine the natural period and natural frequency of the oscillations.
- (d) Determine the directions of the oscillations in the phase plane.
- (e) Using (a)-(d), draw a rough sketch of the phase portrait.