MATH 252: Elementary Differential Equations
Quiz 4

NAME: _____   Date: October 20, 2016

Solve the following problems on this sheet of paper. No calculators or other electronic devices are permitted.

1. (4 points) Rewrite the following linear system in **component** form:

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 0 & \beta \\ \gamma & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

That is, write the ODEs for the components x and y.

2. (6 points) Consider the system

$$\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}, \tag{1}$$

where

$$A = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix}$$

(a) Show that

$$\mathbf{Y}_1(t) = (e^{-3t} - 2e^{-4t}, e^{-3t} - 4e^{-4t}),$$
$$\mathbf{Y}_2(t) = (2e^{-3t} + e^{-4t}, 2e^{-3t} + 2e^{-4t})$$

are both solutions of the above system (1).

(b) Calculate $\mathbf{Y}_1(0)$ and $\mathbf{Y}_2(0)$ and show that they are linearly independent.

(c) Use the above to solve the IVP consisting of system (1) with initial condition

$$\mathbf{Y}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$