MATH 252: Elementary Differential Equations

Quiz 3

NAME: _____ Date: October 6, 2016

Solve the following problems on this sheet of paper. **Note that there is a problem on the back.** No calculators or other electronic devices are permitted.

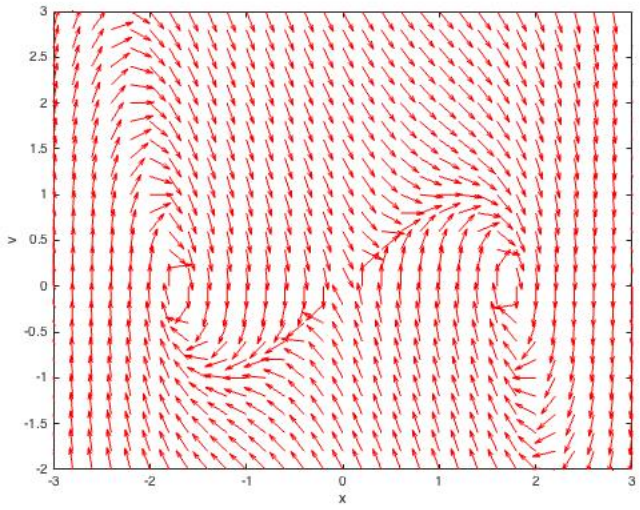
1. (6 points) Consider the second-order differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x + x^3 = 0.$$

(a) Convert this second-order equation into a **first-order system**. That is, identify the vector $\vec{Y}$ and the vector field $F(\vec{Y})$.

(b) Find all equilibrium points of the **system**.

(c) Below is a computer-generated vector field of the system:



Using your answer from part (b), plot the equilibrium points on the above direction field.

(d) Similarly to (c), make a rough sketch of the phase portrait of the system on the direction field.

(e) Briefly (in words) describe the behavior of the solutions.

2. (4 points) Solve the following initial-value problem (IVP):

$$\begin{cases} \frac{dy}{dt} & = \frac{2y}{t} + 2t^2 \\ y(-2) & = 4. \end{cases}$$