Final Exam Info

Date: December 21st, 2016

Location: Normal classroom (TIL-226, Livingston Campus)

Time: 12:00 - 3:00 pm

Notes:

1. The exam will be cumulative, but will be slightly more focused on the material since Exam 2 (non-homogeneous equations and nonlinear systems).
2. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.
3. I will hold extra office hours on Tuesday afternoon. I plan to be in my office from 2-5 pm (Hill 216) on December 20th. If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.
4. There will also be a review held by the TA Michael Breeling on **Friday December 16th, in SEC 210, from 6-8 pm**. He will plan to go over the review problems posted on the website, as well as answer any other questions you might have.
5. I have designed the exam to take approximately two hours, but you may stay the full three if you'd prefer.

Suggestions:

1. Understand all previous exam questions.

1. Read over covered sections in the textbook, as well as notes from class.

2. Understand all assigned homework questions and quizzes. Solutions to quizzes are available on Sakai.

3. Do the review problems posted on the course site.

3. Solve other (unassigned) homework questions from the same section of the textbook.

Material:
All material covered in the course is fair-game for the exam. Regarding the material from the first two exams, I suggest looking at their respective information sheets to get an extensive list of topics to review. For the new material, see the below list and the Course Calendar. As usual, this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for the complete set of topics for the course.

Forcing, or Non-Homogeneous Equations (Sections 4.2-4.3)

(a) Solving second-order non-homogeneous equations with periodic forcing terms (e.g. $y'' + 2y' + 6y = 5\cos(3t)$. Using complexification to write the solution as a single sine or cosine ($y(t) = A\sin(\omega t + \phi)$, i.e. finding A, ω, and phase shift ϕ). Be able to analyze solutions and long-term behavior (e.g plot).

(b) Undamped second-order non-homogeneous equations and resonance. Forcing for the undamped oscillator, where the forcing frequency approaches the natural frequency (beating). Be able to find general solutions, and interpret qualitatively via a graph. Also be able to write the solution as a **product** of sines (or sines and cosines), and be able to find the slow beating frequency (and period) and more rapid frequency from this expression. Also be able to solve the case of resonance (forced frequency equaling the natural frequency) and analyze the corresponding behavior graphically.

Nonlinear systems (Sections 5.1-5.4)

(a) Basic properties of nonlinear systems, including finding equilibrium solutions.

(b) Linearization. Using Jacobian analysis to find the phase portrait of a nonlinear system **locally near an equilibrium point.** Know all of the linear theory (i.e. Chapter 3), including how to draw phase portraits for any 2×2 matrix, and how this relates to the phase portrait for the nonlinear system. In particular, be able to find eigenvalues and eigendirections, as well as directions of rotations for spiral sources/sinks. Note that this picture is only local, and nonlinear systems gennerally don't possess straight-line solutions, although the stable/unstable directions are tangent to the eigendirections at the equilibrium. Also know when the analysis does and does **NOT** apply (*Answer:* Applies when corresponding linear system eigenvalues have **non-zero** real part.)

(c) Nullclines. Be able to plot the nullclines for a two-dimensional nonlinear system. Find the direction of the vector field (i.e. tangent vectors) along the nullcines, and hence in any region in phase space by looking at the boundaries. Combine this with linearization about equilibria to obtain a qualitative picture of the phase portrait. That is, you should be very good at drawing rough approximations to phase portraits for nonlinear systems (rough meaning as accurate as possible). Please practice drawing phase portraits.

(d) Conserved quantities. Be able to show that a given function $H(x, y)$ is conserved for a given system of ODEs. Use this to plot trajectories in the phase plane. And use the actual ODEs to determine the direction of motion along these trajectories.

(e) Hamiltonian systems. Showing that a system is or is not Hamiltonian. If the system is, finding the Hamiltonian and using it to plot the phase portrait (similar to (d) above).

(f) Dissipative systems. Know what is means for a function to be a Lyapunov function for a system of ODEs. Using this to help plot trajectories in the phase plane, as well as the type of an equilibrium point.