

## Exam 2 Info

**Date:** November 17th, 2016

**Location:** Normal classroom (TIL-226, Livingston Campus)

**Time:** In-class (5:00-6:20 pm)

### Notes:

1. Calculators and other electronic devices are **prohibited**. All calculations will be able to be completed by hand.
2. I will hold extra office hours on Wednesday evening. I plan to be in my office from 3-6 pm (Hill 216). If this does not work for you, and you'd like to meet at some other time, please feel free to contact me via email, and I'm sure we can set something up.
3. There will also be a review held by the TA Michael Breeling on **Tuesday November 15th, in Hill 116, from 7-8 pm**. He will plan to go over the review problems posted on the website.

### Suggestions:

1. Read over covered sections in the textbook, as well as notes from class.
2. Understand all assigned homework questions and quizzes. Solutions to quizzes are available on Sakai.
3. Do the review problems posted on the course site.
3. Solve other (unassigned) homework questions from the same section of the textbook.

### Material:

All material up to and including what was covered on Thursday's (November 10th) class (and not covered on Exam 1) is fair-game for the exam. I will finish a few more examples from Section 4.1 on Tuesday, and all of that section will be included on the exam. Thus, Exam 2 will cover all of Chapter 3 (although don't worry about higher-dimensional systems, Section 3.8), and Section 4.1. Some key topics to review are given below. But be aware: this list is **not** exhaustive, and anything covered could appear on the exam. See the Course Calendar on the website for a complete schedule of the material covered.

### Linear Systems (Chapter 3)

- (a) Properties of linear systems. Specifically focus on the Linearity Principle (linear combination of solutions is again a solution, for **linear systems only**) and how to generate solutions of initial-value problems from known solutions using linearity.
- (b) Conversion between second-order equations and first-order linear systems. Know and understand the basic physics of the damped harmonic oscillator, as both a system and an equation.
- (c) Be able to find straight-line solutions of linear systems, if they exist. In other words, you should be very good at finding eigenvalues and corresponding eigenvectors of a matrix  $A$ .

- (d) Finding the general solution of a system from straight-line solutions, and how to solve initial-value problems using row-reduction.
- (e) Know how to solve the cases when a general solution from straight-line solutions do NOT exist. This includes the case of complex and repeated eigenvalues.
- (f) Phase portraits. You should know how to plot phase portraits for **ANY** linear system, based on the eigenvalues and number of linearly independent eigenvectors.
- (g) Know the terminology, in particular, saddles, sinks, sources, spiral sinks, spiral sources, and centers. Know how to determine the direction of rotation for the case of complex and repeated eigenvalues.
- (h) Plotting components of solutions based on the phase portraits and the general solution. As in Exam 1, you should be comfortable translating between phase portraits ( $y$  vs.  $x$ ) and components ( $x$  vs.  $t$ ,  $y$  vs.  $t$ ).
- (i) Solving second-order homogeneous equations. Again, know how to convert between systems and equations, and how the form of solutions for systems produces general solutions for equations.
- (j) Basic properties of the solution of the damped harmonic oscillator, for different physical scenarios. What does overdamped, underdamped, and critically damped mean, and how does it relate to the eigenvalues of the equation?
- (k) Trace-determinant plane. Using this to determine the type of phase portrait your system has. Also, curves traced out in systems with a parameter (i.e.  $A = A(\alpha)$ ), and bifurcation values based on qualitative changes in the phase portrait.
- (l) Matrix exponentials. How to find  $e^{At}$  for any  $2 \times 2$  matrix  $A$ , and how to use it to solve ODEs.

#### Forcing, or Non-Homogeneous Equations (Section 4.1)

- (a) Finding the general solution of a first-order, non-homogeneous equation  $\mathbf{Y}' = A\mathbf{Y} + \mathbf{b}(t)$ , using matrix exponentials.
- (b) Solving second-order non-homogeneous equations ( $ay'' + by' + cy = g(t)$ ) using the method of undetermined coefficients, with forcing functions of the form  $g(t) = e^{at}$  and/or  $g(t) = A_0 + A_1t + \dots + A_nt^n$ . Know how to compute cases when  $g(t)$  **is** and **is not** a solution of the corresponding homogeneous equation.