Matlab Assignment #6

Return of the Slinky

You can refer to the Matlab files from previous assignments to help with the Matlab component of this assignment. Again, for all the Matlab plots you submit, use the function *title* to give the plot a title name which identifies which problem number.

Intrigued by our undulating slinky from Assignment #3, we return to the top of a skyscraper with slinky in hand. We can change the dynamics of our metal toy by attaching various weights to the bottom of the slinky. The slinky's motion can be modeled by the following differential equation:

$$m\frac{d^2y}{d^2t} + 4\frac{dy}{dt} + 2y = 0$$

where m is the mass of the weight in grams and $y(t)$ is the height of the weight in meters relative to its rest position. (In this model, we assume that the mass of the slinky is negligible.)

1. Rewrite the differential equation above as a one-parameter family of linear systems $\frac{d}{dt}\mathbf{Y} = A_m \cdot \mathbf{Y}$ where A_m is a 2×2 matrix which depends on m.

2. Find algebraic formulas for $\mathrm{tr}(A_m)$ and $\det(A_m)$ in terms of the weight's mass m.

3. Either by hand or with Matlab, draw the curve $(\mathrm{tr}(A_m), \det(A_m))$ as well as the parabola $T^2 - 4D = 0$ in the trace$\times$determinant plane. The trace-determinant coordinates of your graph should be $[-5, 5] \times [-5, 5]$ and take $0 \leq m \leq +\infty$. (*Hint: Consider using a change of variable* $s = m^{-1}$.)

4. Identify three points on your curve from Question 3, corresponding to weights with mass 1 gram, 2 grams and 4 grams. Determine the type of each system (saddle, source, sink, center, etc.) and also whether the slinky is overdamped, underdamped, or critically damped.

5. If a harmonic oscillator is underdamped, then the period of its oscillations is given by $4m\pi/\sqrt{4km - b^2}$ where m is the mass, k is the spring constant, and b is the damping coefficient. Use Matlab to plot the period of oscillation for our slinky for the range $2.5 \leq m \leq 25$.

6. If we wanted to use our slinky as a clock, and measure time by counting the number of complete oscillations, would it be better to add a very large weight or a very small weight to the end of our slinky? Explain your answer.

7. Suppose we attach a weight whose mass is 2 grams to the end of the slinky. Find the particular solution $y(t)$ given the initial condition $y(0) = 2$ and $y'(0) = 0$. (To solve this question, you can use the matrix exponential method, one of the methods from sections 3.3-3.5, or the method of the lucky guess).

8. Use Matlab to graph the position of the slinky – $y(t)$ – using the solution you found in Question 7 for the time period $0 \leq t \leq 10$.